ELECTRIC RESISTIVITY - MODELLING OF

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OF $Fe_{50}Co_{50}$ THIN LAYERS - EXPERIMENTAL RESULTS

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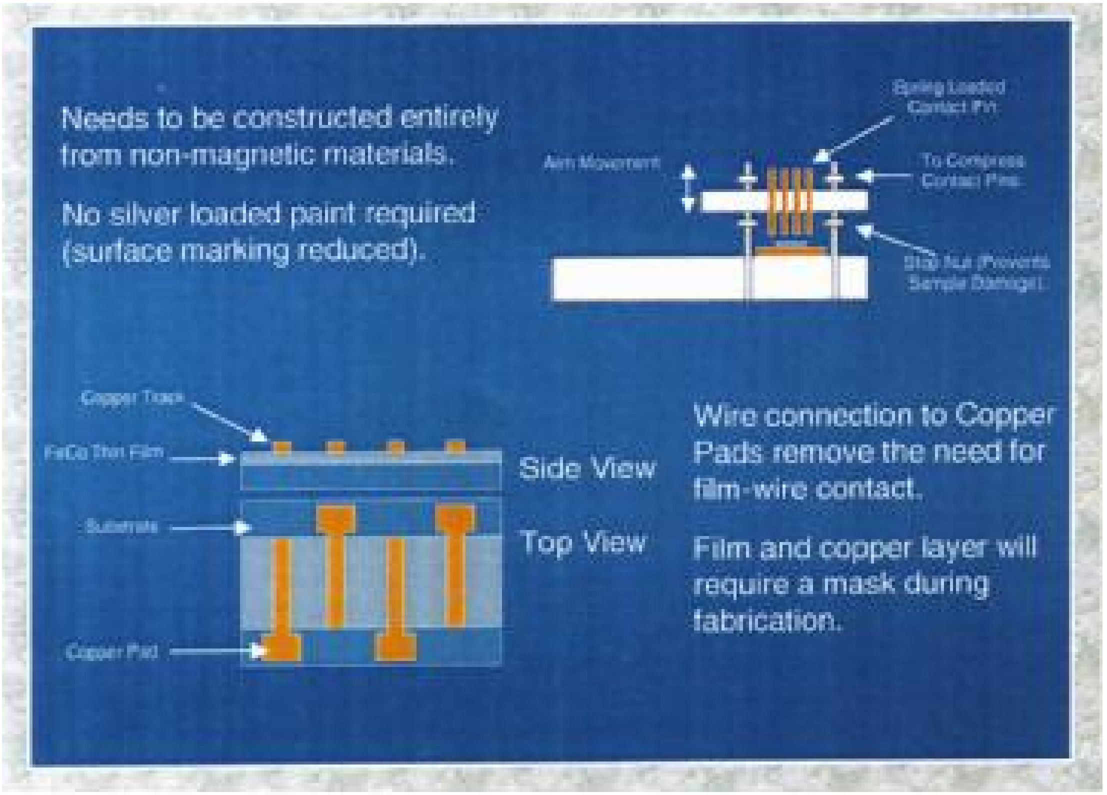
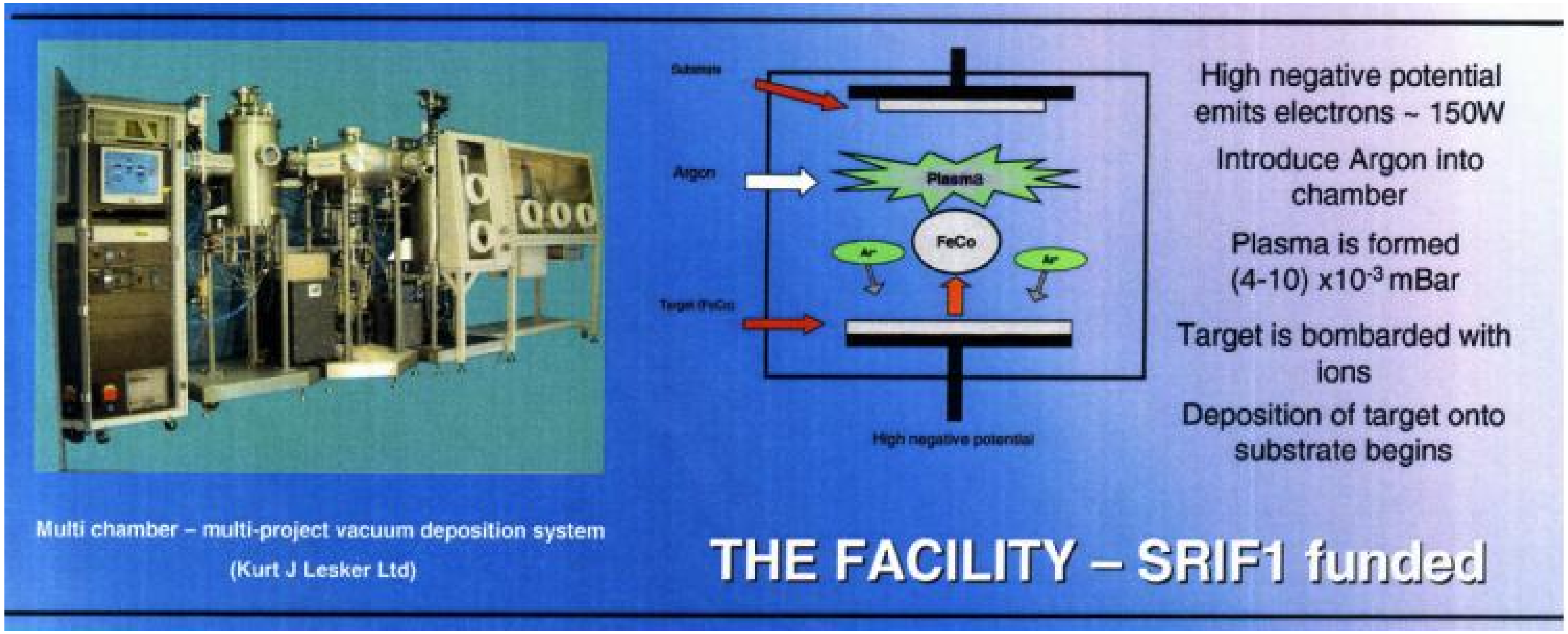
Experimental results

In this work the resistivity *ρ* of sputtered $Fe_{50}Co_{50}$/Si thin films has been studied as a function of magnetic layer thickness. Experimental data obtained with four point method reveal a very fast increase of the resistivity for $Fe_{50}Co_{50}$ thickness *a* < 20 nm (Fig. 1.)



Sputtering system



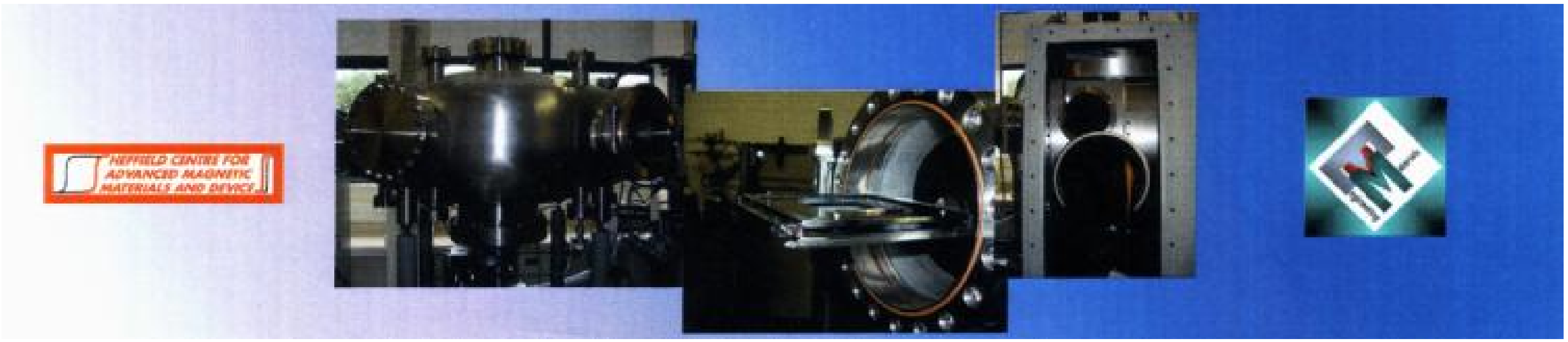
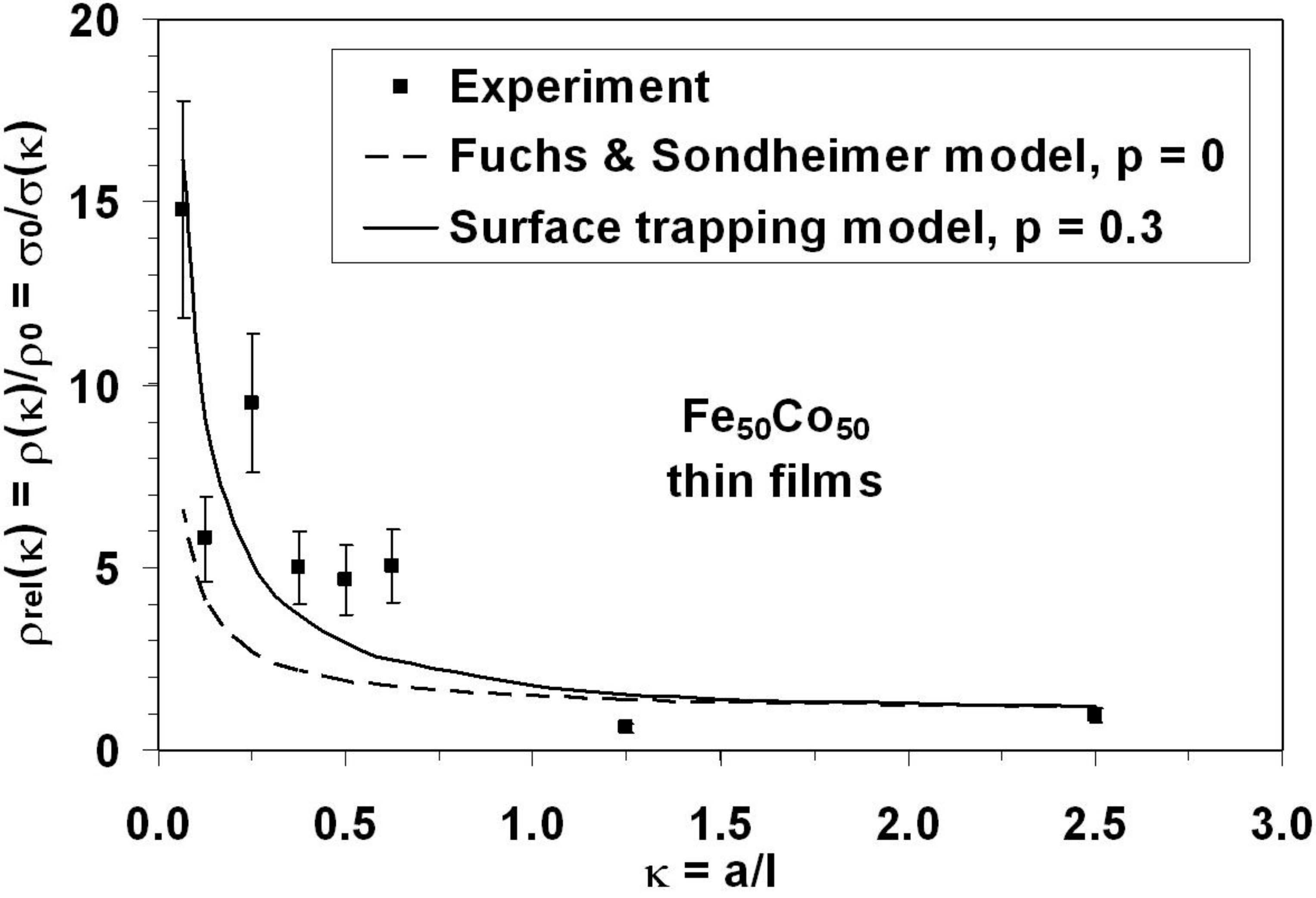


Fig. 1.



20
15
10
5
0
ρrel(κ) = ρ(κ)/ρ0 = σ0/σ(κ)
Experiment
Fuchs & Sondheimer model, p = 0
Surface trapping model, p = 0.3
Fe50Co50
thin films
0.0
0.5
1.0
1.5
2.0
2.5
3.0
κ = a/l

Fuchs & Sondheimer model

100

Proc. Cambridge Philos. Soc.
34 (1938) 100

THE CONDUCTIVITY OF THIN METALLIC FILMS ACCORDING TO THE ELECTRON THEORY OF METALS

By K. FUCHS, H. H. Wills Physical Laboratory, University of Bristol

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The Mean Free Path of Electrons in Metals

By E. H. Sondheimer*,
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Fuchs & Sondheimer model

1.5. Many approximate theoretical treatments have been given of the various free-path phenomena, but we shall not refer to these (except where they are particularly relevant to the discussion), as they have been superseded by the strict statistical analysis based on the Boltzmann equation for the distribution function of the conduction electrons†. This equation is formed by equating the rate of change in f due to external fields to the rate of change due to the collision mechanism, which is assumed to be given by equation (3). In the presence of an electric field $\mathbf{E}$ and a magnetic field $\mathbf{H}$, the Boltzmann equation for quasi-free electrons takes the form

$$-\frac{\epsilon}{m}\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{H}\right).\operatorname{grad}_{\mathbf{v}} f+\mathbf{v}.\operatorname{grad}_{\mathbf{r}} f=-\frac{f-f_0}{\tau}, \quad . \quad . \quad (5)$$

* Gaussian units are used throughout the present article.

† The use of the Boltzmann equation in this connection appears to have been first suggested by Peierls.

which is purely classical except that the mass m is to be regarded as an effective mass, while the equilibrium distribution function is the Fermi–Dirac function

$$f_0(E)=\frac{1}{e^{(E-\zeta)/kT}+1} \quad . \quad . \quad . \quad (6)^*$$

The time of relaxation τ is supposed to depend on the absolute value of $\mathbf{v}$ only.

The main problem, then, is to solve equation (5) for the various cases of interest, and to use the solution to calculate the current density $\mathbf{J}$ by means of the usual formula

$$\mathbf{J}=-2\epsilon\left(\frac{m}{h}\right)^3\int \mathbf{v} f\, d\mathbf{v}. \quad . \quad . \quad . \quad (7)$$

2.12. A somewhat more general theory, which does not assume that the scattering at the surface of the film is entirely diffuse, can be obtained as follows. We assume that a fraction p of the electrons is scattered elastically at the surface with reversal of the velocity component v_z, while the rest are scattered diffusely with complete loss of their drift velocity. p is supposed to be a constant independent of the direction of motion of the electrons. This is of course a highly artificial model, which in effect merely interpolates between the extreme cases of perfectly diffuse reflection, considered above, and perfectly specular reflection, for which the conductivity is unaltered. However, in the absence of any detailed theory of the nature of the surface scattering mechanism it is best to work with the simplest possible assumptions.

The distribution function of the electrons leaving the surface $z=0$ is now given by

$$f_0+f_1^+(v_z, z=0)=p\{f_0+f_1^-(-v_z, z=0)\}+(1-p)f_0, \quad . \quad . \quad (22)$$

and similarly, at $z=a$,

$$f_0+f_1^-(v_z, z=a)=p\{f_0+f_1^+(-v_z, z=a)\}+(1-p)f_0. \quad . \quad . \quad (23)$$

These equations are sufficient to determine $F(\mathbf{v})$, and instead of (11) we obtain for the distribution function

$$\left.\begin{aligned} f_1^+(\mathbf{v}, z)&=\frac{\epsilon\tau E}{m}\frac{\partial f_0}{\partial v_x}\left\{1-\frac{1-p}{1-p\exp(-a/\tau v_z)}\exp\left(-\frac{z}{\tau v_z}\right)\right\} \quad (v_z>0),\\ f_1^-(\mathbf{v}, z)&=\frac{\epsilon\tau E}{m}\frac{\partial f_0}{\partial v_x}\left\{1-\frac{1-p}{1-p\exp(a/\tau v_z)}\exp\left(\frac{a-z}{\tau v_z}\right)\right\} \quad (v_z<0). \end{aligned}\right\} \quad . \quad . \quad . \quad (24)$$

Fuchs & Sondheimer model

κ							
	2.50	1.172838	100	233.5241	0.934097	0.05699738	
	1.25	1.366360	50	149.1658	0.596663	0.59243313	
	0.63	1.739735	25	1258.0479	5.032191	10.8402665	
	0.50	1.916145	20	1163.8972	4.655589	7.50455441	
	0.38	2.197449	15	1246.2012	4.984805	7.76935226	
	0.25	2.724012	10	2370.4672	9.481869	45.6686239	
	0.13	4.134033	5	1443.9107	5.775643	2.69488441	
	0.06	6.594960	2.5	3697.7024	14.79081	67.1719502	142.2991

$\kappa_i := in0$ $\quad\quad p := in1$ $\quad\quad p = 0$

$$\Phi(\kappa) := \left[\frac{1}{\kappa} - \frac{3}{2\cdot\kappa^2}\cdot(1-p)\cdot\int_1^{\infty}\left(\frac{1}{t^3}-\frac{1}{t^5}\right)\cdot\frac{1-e^{-\kappa\cdot t}}{1-p\cdot e^{-\kappa\cdot t}}\,dt\right]^{-1}$$

$\rho_{rel}(\kappa) = \rho(\kappa)/\rho_0 = \sigma_0/\sigma(\kappa)$ $\quad\quad \kappa = a/l$

$$\rho_{rel}(\kappa) := \frac{\Phi(\kappa)}{\kappa}$$

$\rho_{rel}(0.01) = 26.431$

```
out0 := | i ← 0
        | for κ ∈ κi
        |     | ρrel_i ← ρrel(κ)
        |     | i ← i + 1
        | ρrel
```

ρo =	250	Ω*m	l=	40	nm	p =	0

Thomson's model

2. J. J. Thomson's formula

The first formula was given in 1901 by Thomson†. To obtain this formula we make the following assumptions.

(α) When an electron collides with the surface of the film, the probability that it is scattered into any solid angle $d\omega$ is $d\omega/2\pi$, and is thus independent of the initial and final directions of the motion of the electron.

(β) The free path of an electron in the bulk metal is a constant λ_0.

We assume further that λ_0 is greater than the film thickness t.

Consider an electron which starts from a point P at a distance z from the surface

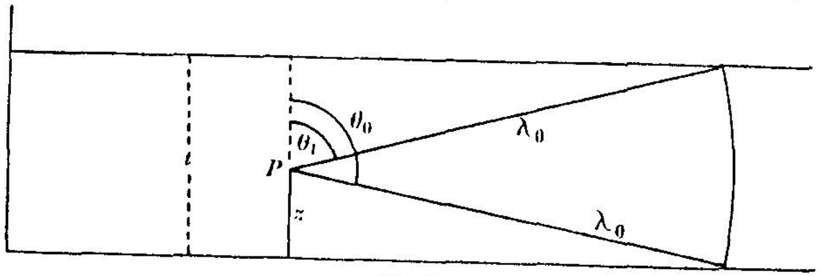

Fig. 1.

of the film, and moves in a direction making an angle θ with the z-axis (cf. Fig. 1). Its free path λ is given by

$$\lambda = \begin{cases} (t-z)/\cos\theta & (0 \leqslant \theta \leqslant \theta_1), \\ \lambda_0 & (\theta_1 \leqslant \theta \leqslant \theta_0), \\ -z/\cos\theta & (\theta_0 \leqslant \theta \leqslant \pi), \end{cases} \quad (1)$$

where $\cos\theta_1 = (t-z)/\lambda_0, \quad \cos\theta_0 = -z/\lambda_0.$

The mean free path is obtained by taking the mean value of λ over all angles θ and all distances z. We obtain

$$\bar{\lambda} = \frac{1}{2t}\int_0^t dz \int_0^\pi \lambda \sin\theta \, d\theta$$

$$= \frac{t}{2}\left[\log\frac{\lambda_0}{t} + \frac{3}{2}\right]. \quad (2)$$

Since σ is proportional to the mean free path, we obtain for the ratio of the conductivity σ to the conductivity σ_0 of the bulk metal

$$\frac{\sigma}{\sigma_0} = \frac{t}{2\lambda_0}\left[\log\frac{\lambda_0}{t} + \frac{3}{2}\right], \quad (3)$$

which is Thomson's formula.

The assumptions used in deriving this formula are incorrect for the following reasons.

(a) To obtain the mean free path we must consider all the electrons in the metal at a given moment and average over their free paths; it is not correct to take the mean of all the free paths of a given electron, as is done above.

(b) Free paths starting on the surface of the film are neglected.

(c) The statistical distribution of the free paths about λ_0 in the bulk metal is neglected.

Surface trapping model

Extension of Thomson model

In the case of significant surface roughness we can expected additional electrons scattering. Within the simplest approach to this problem one can assume that effective thickness of the film is reduced by the RMS value of vertical roughness (Root of Mean Square). This approximation is justified when vertical (perpendicular) and planar (in-plain) dimensions of "roughness islands" are smaller then mean free paths (ballistic regime).

A roughness can both change the effective thickness of the film and traps electrons excluding them for some time from current conducting process. If the distance from starting position of the electron to the point of collision with rough surface is less than mean free path (of bulk) such electron is switched of from conduction process. When we assume that only definite fraction p of electrons scattered by interface comes back immediately to the roughness-free region of the film, we can modify previous models and obtain following formulae for the resistivity:

for $\kappa < 1$

$$\rho_{\mathrm{rel}}(\kappa) = \frac{1}{\kappa}\frac{2}{1 + p\left(\frac{1}{2} + \ln\frac{1}{\kappa}\right)}$$

for $\kappa > 1$

$$\rho_{\mathrm{rel}}(\kappa) = \frac{2}{2 - \left(1 - \frac{1}{2}p\right)\frac{1}{\kappa}}$$

Conclusions

The good accordance with experimental data (Fig. 1.) has been achieved for *p* = 0.3 (1-*p*: trapping coefficient) and *l* = 40 nm within surface trapping model.

We also suggest the enhancement of our model taking into account quantum effects, which are important when thickness of films or mean size of surface roughness is comparable with Fermi - de Broglie wave length of electrons [3,4,5].

The studied high magnetostrictive $Fe_{50}Co_{50}$ films are promising for production of sensors, actuators and MagMEMS devices [6].

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