SPIN ACCUMULATION DYNAMICS IN A BI-LAYER WITH ROTATING MAGNETIZATION

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We theoretically investigate spin accumulation in a bi-layer consisting of two ferromagnets, the magnetization of one of which rotates. It is assumed that the rotation of the layer magnetization is caused by an outside source which has fixed frequency. The magnetization of the second layer is fixed. The charge and spin currents are described within the linear response by the equation for diffusive transport. The generalized Landau-Lifshits-Gilbert equation is chosen as the equation of motion for spin accumulation [1]. The rotation of the magnetization leads to the oscillation of the spin-polarized current in both layers. We have found stationary solution for charge and spin accumulation. In the stationary state the current oscillation in the layer with rotating magnetization is a superposition of the harmonics which are multiples of the magnetization rotation frequency and of the time-independent term. The same is true for the charge and spin current in the second layer. The amplitudes of the harmonics are different in the two layers. The dependence of the amplitudes on diffusion coefficient and spin relaxation time is investigated. The components of magnetization current rotate due to spin accumulation.

To estimate the time of relaxation of the system to the stationary state we consider an interaction between spin-polarized current and rotating magnetization of a single layer. A closed analytical solution for initial-value problem is constructed. The exponentially decayed terms give the estimation of relaxation time to stationary state.

SINGLE LAYER

BASIC EQUATIONS

$$j_e^x = 2C_0 E^x(x) - 2D_0 \frac{\partial n^0(x,y)}{\partial x} - 2\vec{D}\frac{\partial \vec{m}(x,y)}{\partial x}$$

$$\vec{C} = \beta C_0 \vec{M}_d, \quad \vec{D} = \beta' D_0 \vec{M}_d \qquad \lambda_{sf} = \sqrt{2D_0 \tau_{sf}}, \quad \lambda_J = \sqrt{2\hbar D_0 / J}$$

$$j_e^y = 2C_0 E^y(x) - 2D_0 \frac{\partial n^0(x,y)}{\partial y} - 2\vec{D}\frac{\partial \vec{m}(x,y)}{\partial y}$$

$$\frac{d\vec{m}}{dt} + div\, \vec{j}_m + \frac{J}{\hbar}\left[\vec{m}\,\vec{M}_d\right] = -\frac{\vec{m} - \alpha \vec{M}_d}{\tau_{sf}}$$

$$\vec{j}_m^{\,x} = 2\vec{C}E^x(x) - 2\vec{D}\frac{\partial n^0(x,y)}{\partial x} - 2D_0 \frac{\partial \vec{m}(x,y)}{\partial x}$$

$$\vec{M}_d = \{0, M_d \cos(\omega t), M_d \sin(\omega t)\}$$

$$\vec{j}_m^{\,y} = -2\vec{D}\frac{\partial n^0(x,y)}{\partial y} - 2D_0 \frac{\partial \vec{m}(x,y)}{\partial y}$$

$$\vec{s} = \vec{m} - \alpha \vec{M}_d$$

$$\begin{pmatrix} \frac{1}{2D_0}\frac{d}{dt} - \Delta_{y,z} + \frac{1}{\lambda_{sf}^2} & \frac{1}{\lambda_J^2} M_d \sin(\omega t) & -\frac{1}{\lambda_J^2} M_d \cos(\omega t) \\ -\frac{1}{\lambda_J^2} M_d \sin(\omega t) & \frac{1}{2D_0}\frac{d}{dt} - \Delta_{y,z} + \frac{1}{\lambda_{sf}^2} + \beta\beta' M_d^2 \cos^2(\omega t)\Delta_{y,z} & \beta\beta' M_d^2 \sin(\omega t)\cos(\omega t)\Delta_{y,z} \\ \frac{1}{\lambda_J^2} M_d \cos(\omega t) & \beta\beta' M_d^2 \sin(\omega t)\cos(\omega t)\Delta_{y,z} & \frac{1}{2D_0}\frac{d}{dt} - \Delta_{y,z} + \frac{1}{\lambda_{sf}^2} + \beta\beta' M_d^2 \sin^2(\omega t)\Delta_{y,z} \end{pmatrix} \times \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\omega\gamma M_d \sin(\omega t)}{2D_0} \\ -\frac{\omega\gamma M_d \cos(\omega t)}{2D_0} \end{pmatrix}$$

STATIONARY SOLUTION

$$\vec{s} = \begin{pmatrix} \sum_m \left(A_m^{(1)} \cos(m\omega t) + B_m^{(1)} \sin(m\omega t)\right) \\ \sum_m \left(A_m^{(2)} \cos(m\omega t) + B_m^{(2)} \sin(m\omega t)\right) \\ \sum_m \left(A_m^{(3)} \cos(m\omega t) + B_m^{(3)} \sin(m\omega t)\right) \end{pmatrix} + \vec{s}_0(t)$$

RELAXATION

$$\vec{s}_0(t) = \exp\left(-\frac{2D_0}{\lambda_{sf}^2} t\right) \cdot \vec{g}^{(0)}(t)$$

$$\lambda_{sf}^2 / 2D_0 = \tau_{sf} \cong 10^{-11} \sec$$

$$g_x^{(0)}(t) = \frac{2D_0 C_z M_d}{\lambda_J^2} \frac{1}{\sqrt{\omega^2 + \left(4D_0^2 M_d^2/\lambda_J^4\right)}} \sin\left(\sqrt{\omega^2 + \left(4D_0^2 M_d^2/\lambda_J^4\right)}\, t\right) -$$

$$\frac{2D_0 C_y M_d\, \omega}{\lambda_J^2} \frac{1}{\omega^2 + \left(4D_0^2 M_d^2/\lambda_J^4\right)} \left[1 - \cos\left(\sqrt{\omega^2 + \left(4D_0^2 M_d^2/\lambda_J^4\right)}\, t\right)\right]$$

Coordinate-independent part:

Kronecker delta

$$A_m^{(1)} = -\frac{\left(\omega\gamma M_d^2/2\lambda_J^2 D_0\right) \cdot \left(-(m\omega/2D_0)^2 + \left(M_d^2/\lambda_J^4\right) + \left(1/\lambda_{sf}^4\right)\right) \cdot \Delta_m}{(m\omega/2D_0)^4 + \left(\left(M_d^2/\lambda_J^4\right) + \left(1/\lambda_{sf}^4\right)\right)^2 + 2 \cdot (m\omega/2D_0)^2 \cdot \left(-\left(M_d^2/\lambda_J^4\right) + \left(1/\lambda_{sf}^4\right)\right)}$$

$$B_m^{(1)} = -\frac{(m\omega/2D_0) \cdot \left(\omega\gamma M_d^2/D_0 \lambda_J^2 \lambda_{sf}^2\right) \cdot \Delta_m}{(m\omega/2D_0)^4 + \left(\left(M_d^2/\lambda_J^4\right) + \left(1/\lambda_{sf}^4\right)\right)^2 + 2 \cdot (m\omega/2D_0)^2 \cdot \left(-\left(M_d^2/\lambda_J^4\right) + \left(1/\lambda_{sf}^4\right)\right)}$$

TWO LAYERS

Charge and spin currents:

$$j_e^x = 2C_0 E - 2D_0 \frac{\partial n}{\partial x} - 2\beta' D_0 \vec{M}_d \frac{\partial \vec{m}}{\partial x} \qquad \vec{j}_m^{\,x} = 2\beta C_0 \vec{M}_d E - 2\beta' D_0 \vec{M}_d \frac{\partial n}{\partial x} - 2D_0 \frac{\partial \vec{m}}{\partial x}$$

Equations of motion for charge and spin accumulation:

$$\frac{\partial n}{\partial t} = -\frac{\partial j_e^x}{\partial x} \qquad \frac{\partial \vec{m}}{\partial t} + \frac{\partial \vec{j}_m}{\partial x} + \frac{J}{\hbar}\left[\vec{m}\vec{M}_d\right] = -\frac{\vec{m} - \alpha \vec{M}_d}{\tau_{sf}}$$

Right layer: $\vec{M}_d = \{0, M_d \cos\omega t, M_d \sin\omega t\}$ **Left layer:** $\vec{M}_d = \{0, 0, M_d\}$

Right layer:

$$\begin{pmatrix} \frac{\partial}{\partial t} - 2D_0 \frac{\partial^2}{\partial x^2} & 0 & -2\beta' D_0 \left(M_d \cos\omega t\right)\frac{\partial^2}{\partial x^2} & -2\beta' D_0 \left(M_d \sin\omega t\right)\frac{\partial^2}{\partial x^2} \\ 0 & \frac{\partial}{\partial t} - 2D_0 \frac{\partial^2}{\partial x^2} + \frac{1}{\tau_{sf}} & \frac{J}{\hbar} M_d \sin\omega t & -\frac{J}{\hbar} M_d \cos\omega t \\ -2\beta' D_0 \left(M_d \cos\omega t\right)\frac{\partial^2}{\partial x^2} & -\frac{J}{\hbar} M_d \sin\omega t & \frac{\partial}{\partial t} - 2D_0 \frac{\partial^2}{\partial x^2} + \frac{1}{\tau_{sf}} & 0 \\ -2\beta' D_0 \left(M_d \sin\omega t\right)\frac{\partial^2}{\partial x^2} & \frac{J}{\hbar} M_d \cos\omega t & 0 & \frac{\partial}{\partial t} - 2D_0 \frac{\partial^2}{\partial x^2} + \frac{1}{\tau_{sf}} \end{pmatrix} \times \begin{pmatrix} n \\ m_x \\ m_y \\ m_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{\alpha M_d \cos\omega t}{\tau_{sf}} \\ \frac{\alpha M_d \sin\omega t}{\tau_{sf}} \end{pmatrix}$$

TRANSFORMATION $\hat{H} \rightarrow \hat{S}\hat{H}\hat{S}^{-1}, \quad g \rightarrow \hat{S}g$ REMOVES TIME DEPENDENCE

$$\begin{pmatrix} \frac{\partial}{\partial t} + a_1 & 0 & u_1 & 0 \\ 0 & \frac{\partial}{\partial t} + b_1 & 0 & -u_2 \\ u_1 & 0 & \frac{\partial}{\partial t} + b_1 & -\omega \\ 0 & u_2 & \omega & \frac{\partial}{\partial t} + b_1 \end{pmatrix} \times \begin{pmatrix} \breve{g}_0 \\ \breve{g}_x \\ \breve{g}_y \\ \breve{g}_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{\gamma M_d}{\tau_{sf}} \\ 0 \end{pmatrix}$$

$$\breve{g}^r(t,x) = \breve{g}^r_{[n]} + \breve{g}^{r,\{0\}}(t,x)$$

$$\left(\hat{I}\frac{\partial}{\partial t} + \gamma^r \frac{\partial^2}{\partial x^2} + b^r \right) \breve{g}^r_{(h)}(t,x) = 0$$

$$g^r(t,x) = \int_{-\infty}^{\infty} \sum_{j=1}^{4} \left(\alpha^r_j(\Omega)\exp[iq^r_j(\Omega)x] + \beta^r_j(\Omega)\exp[-iq^r_j(\Omega)x] \right) e^{i\Omega t}\, d\Omega + (b^r)^{-1}M^r$$

Solution of the homogeneous equation

$$\begin{pmatrix} \alpha^r_{1j}(\Omega) \\ \alpha^r_{2j}(\Omega) \\ \alpha^r_{3j}(\Omega) \\ \alpha^r_{4j}(\Omega) \end{pmatrix} \rightarrow \begin{pmatrix} f^r_{\alpha j}(\Omega)\cdot\alpha^r_{1j}(\Omega) \\ f^r_{\alpha j}(\Omega)\cdot\alpha^r_{2j}(\Omega) \\ f^r_{\alpha j}(\Omega)\cdot\alpha^r_{3j}(\Omega) \\ f^r_{\alpha j}(\Omega)\cdot\alpha^r_{4j}(\Omega) \end{pmatrix}, \qquad \begin{pmatrix} \beta^r_{1j}(\Omega) \\ \beta^r_{2j}(\Omega) \\ \beta^r_{3j}(\Omega) \\ \beta^r_{4j}(\Omega) \end{pmatrix} \rightarrow \begin{pmatrix} f^r_{\beta j}(\Omega)\cdot\beta^r_{1j}(\Omega) \\ f^r_{\beta j}(\Omega)\cdot\beta^r_{2j}(\Omega) \\ f^r_{\beta j}(\Omega)\cdot\beta^r_{3j}(\Omega) \\ f^r_{\beta j}(\Omega)\cdot\beta^r_{4j}(\Omega) \end{pmatrix}, \quad j = 1,2,3,4$$

Similar equations were obtained for the left layer

MATCHING AT THE INTERFACE:

$$\mathbf{L}_0^r(\Omega)F^r(\Omega)+\mathbf{L}_+^r(\Omega+\omega)F^r(\Omega+\omega)+\mathbf{L}_-^r(\Omega-\omega)F^r(\Omega-\omega)+$$

$$P_0^r(\Omega)\delta(\Omega)+P_+^r(\Omega+\omega)\delta(\Omega+\omega)+P_-^r(\Omega-\omega)\delta(\Omega-\omega)=\mathbf{L}^l(\Omega)F^l(\Omega)+P_0^l(\Omega)\delta(\Omega)$$

stationary solution: $\mathbf{F}^r=\sum f_n^r\delta(\Omega-n\omega),\quad \mathbf{F}^l=\sum f_n^l\delta(\Omega-n\omega)$

recurrent equations

$$\mathbf{L}_0^r(0)f_0^r+\mathbf{L}_+^r(\omega)f_1^r+\mathbf{L}_-^r(-\omega)f_{-1}^r+P_0^r=\mathbf{L}_0^l(0)f_0^l+P_0^l \qquad \delta(\Omega)$$

$$\mathbf{L}_0^r(\omega)f_1^r+\mathbf{L}_+^r(2\omega)f_2^r+\mathbf{L}_-^r(0)f_0^r+P_-^r=\mathbf{L}_0^l(\omega)f_1^l \qquad \delta(\Omega-\omega)$$

$$\mathbf{L}_0^r(2\omega)f_2^r+\mathbf{L}_+^r(3\omega)f_3^r+\mathbf{L}_-^r(\omega)f_1^r=\mathbf{L}_0^l(2\omega)f_2^l \qquad \delta(\Omega-2\omega)$$

$$\mathbf{L}_0^r(2\omega)f_2^r+\mathbf{L}_+^r(3\omega)f_3^r+\mathbf{L}_-^r(\omega)f_1^r=\mathbf{L}_0^l(2\omega)f_2^l \qquad \delta(\Omega-2\omega)$$

$$\mathbf{L}_0^r(-2\omega)f_{-2}^r+\mathbf{L}_+^r(-\omega)f_{-1}^r+\mathbf{L}_-^r(-3\omega)f_{-3}^r=\mathbf{L}_0^l(-2\omega)f_{-2}^l \qquad \delta(\Omega+2\omega)$$

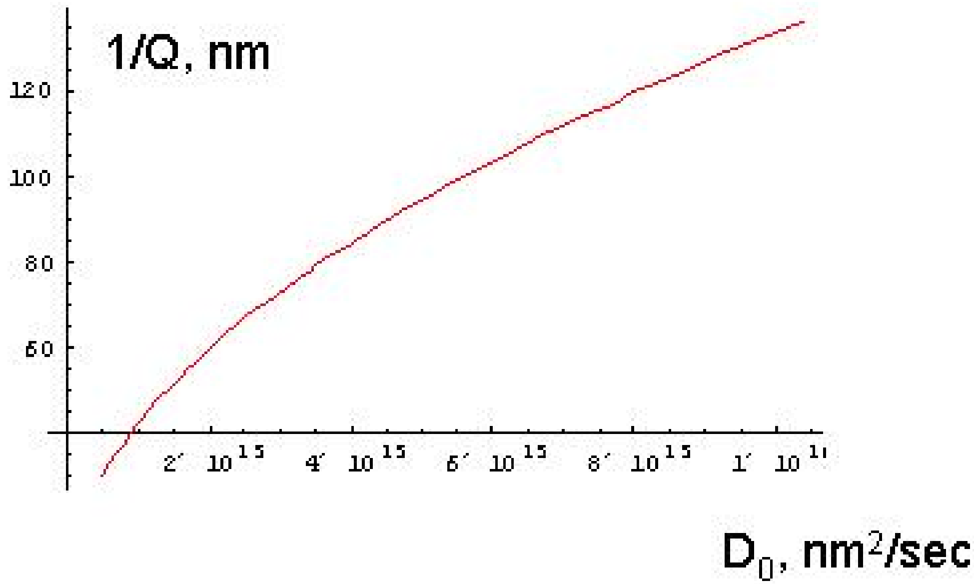
Small n: $f_1^r / f_0^r \sim 10^{-4}$

Q^{-1} determines characteristic length of changing of spin and charge current changing

$$\tau_{sf} = 0.5 \cdot 10^{-12}\,\text{sec}$$

$$1\,Hz < \omega < 200\,Hz$$

$$h/J \cong 10^{-12}\,\text{sec}$$



Large n: $Q \sim -\sqrt{\Omega/D_0}\,(1-i)$

$$f_n^l, f_n^r \sim \lambda^n, |\lambda| < 1$$

[1] S. Zhang, P. M. Levy, A. Fert, PRL **88**, 236601 (2002)
[2] A. Shpiro, P. M. Levy, S. Zhang, PRB **67**, 104430 (2003)
[3] A. Persova, A. Vedyayev, et al., *submitted*