# ANOMALOUS HALL EFFECT IN IV - VI SEMICONDUCTORS

*Anna Dyrdał*[1], *Vitalii Dugaev*[2,3,4] *Józef Barnaś*[1,5], *Beata Brodowska*[6], *Witold Dobrowolski*[6]

[1]Department of Physics, Adam Mickiewicz University, Poznań, Poland
[2]Department of Physics, Rzeszów University of Technology, Rzeszów, Poland
[3]Department of Physics and CFIF, Instituto Superior Técnico, Technical University of Lisbon, Lisbon, Portugal
[4]Frantsevich Institute for Problems Materials Science, National Academy of Sciences of Ukraine, Chernovtsy, Ukraine
[5]Institute of Molecular Physics, Polish Academy of Sciences, Poznań, Poland
[6]Institute of Physics, Polish Academy of Sciences, Warsaw, Poland

## *Abstract*

In ferromagnetic metals and magnetic semiconductors, the off-diagonal conductivity is proportional to the external magnetic field, which is connected with ordinary Hall effect, and an additional term which is proportional to the magnetization of the sample and does not disappear at zero magnetic field. This extraordinary term is known as the anomalous Hall effect (AHE). The origin of the AHE is the spin-orbit interaction in the presence of spin polarization. There are two groups of mechanisms that are responsible for AHE: so called extrinsic mechanisms (skew scattering and side jump) and intrinsic mechanisms which are related to the topology of electron energy bands.

We consider narrow-gap IV-VI magnetic semiconductors where the relativistic terms are not small and determine both the non-parabolicity of the energy spectrum and strong spin-orbit interaction. We use the relativistic Dirac model and the theory of linear response to calculate the topological contribution to the off-diagonal anomalous Hall conductivity. We also present some experimental data.

## *Relativistic Dirac Model*

The matrix form of the Hamiltonian is as follows:

$$H = \begin{pmatrix} \Delta - g_c M\sigma_z & v_0\boldsymbol{\sigma}\cdot\mathbf{k} \\ v_0\boldsymbol{\sigma}\cdot\mathbf{k} & -\Delta - g_v M\sigma_z \end{pmatrix}, \tag{1}$$

To calculate the topological contribution we start from the general Kubo formula for $\omega \neq 0$

$$\sigma_{xy}(\omega) = \frac{e^2\hbar}{\omega}\,\mathrm{Tr}\int\frac{d\varepsilon}{2\pi}\,\frac{d^3k}{(2\pi)^3}\,v_x\,G_{\mathbf{k}}(\varepsilon+\omega)\,v_y\,G_{\mathbf{k}}(\varepsilon) \tag{2}$$

where:

$$v_x = \frac{v_0}{\hbar}\begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix}, \quad v_y = \frac{v_0}{\hbar}\begin{pmatrix} 0 & \sigma_y \\ \sigma_y & 0 \end{pmatrix}.$$

## *Contribution to the AHE conductivity from the Fermi surface*

First, we calculate contribution to the anomalus Hall conductivity from states on the Fermi level. The Kubo formula in this case may be written in the following form:

$$\sigma_{xy} = \frac{e^2 v_0^2}{2\pi\hbar}\,\mathrm{Tr}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix} G^R_{\mathbf{k}}(\varepsilon)\begin{pmatrix} 0 & \sigma_y \\ \sigma_y & 0 \end{pmatrix} G^A_{\mathbf{k}}(\varepsilon) \tag{3}$$

Calculating the above integral, we get the expression for the Hall conductivity:

$$\sigma_{xy} = -\frac{3Me^2(2g\varepsilon - g^*\Delta)}{16\pi^2 v_0\hbar\left(\varepsilon^2-\Delta^2\right)^{1/2}} \tag{4}$$

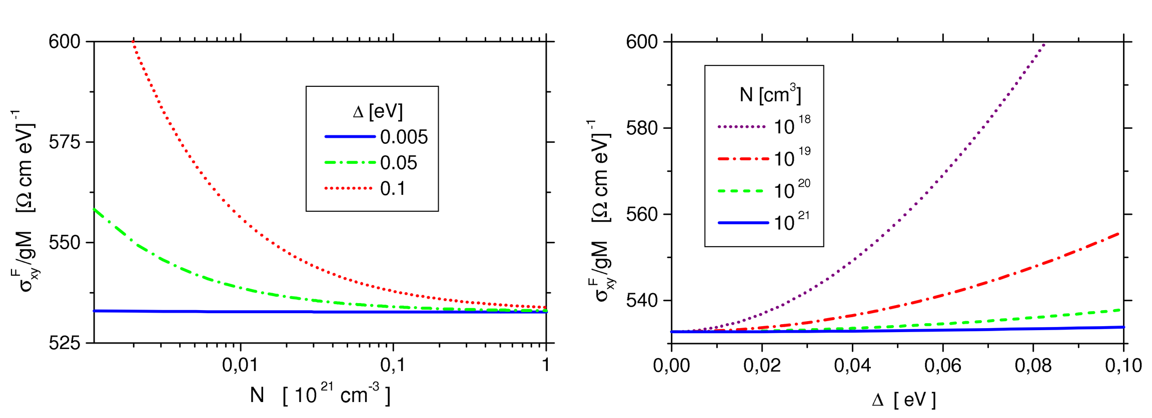


FIGURE 1: Hall conductivity from states at the Fermi surface in the case when the Fermi level is in the valence band as a function of the paramter $\Delta$ and impurity concentration $N$. The other parameters: $v_0 = 5\times10^{-8}$ eV cm, $g_c = g_v = g$

## *Topological contribution to the AHE*

The topological contribution may be calculated within the Kubo formula (2). After calculating the trace in eq. (2) and taking the limit $\omega, M \to 0$ we get:

$$\sigma_{xy} = \frac{4ie^2v_0^2gM}{\hbar}\int\frac{d\varepsilon}{2\pi}\,\frac{d^3k}{(2\pi)^3}\,\frac{P_0(\varepsilon+\mu)}{[\varepsilon-\epsilon_{0\mathbf{k}}+\mu+i\eta\,\mathrm{sgn}\,\varepsilon]^4\,[\varepsilon+\epsilon_{0\mathbf{k}}+\mu+i\eta\,\mathrm{sgn}\,\varepsilon]^4} \tag{5}$$

where $P_0(\varepsilon) = dP(\varepsilon_1,\varepsilon_2)/d\varepsilon_2\,|_{\varepsilon_1,\varepsilon_2=\varepsilon}$is nonzero component of the Taylor expansion of $P(\varepsilon_1,\varepsilon_2)$ which is equal to the trace of the numerator in eq. (2)$(\varepsilon_1 = \epsilon+\mu+\omega,\ \varepsilon_2 = \epsilon+\mu)$ and $\epsilon_{0\mathbf{k}} = \pm\left(\Delta^2+v_0^2k^2\right)^{1/2}$ is the energy spectrum of the valence / conduction band in the limit of $M=0$.

After integrating over $\varepsilon$ we obtain:

$$\sigma_{xy} = \sigma^v_{xy} + \sigma^c_{xy} \tag{6}$$

where $\sigma^\alpha_{xy}$ $(\alpha = v,c)$ is contribution from the valence and conduction bands

$$\sigma^\alpha_{xy} = -\frac{2e^2v_0^2gM}{3\hbar}\int\frac{d^3k}{(2\pi)^3}\,f(E^0_{\alpha\mathbf{k}})\left[\frac{P_0'''(E^0_{\alpha\mathbf{k}})}{(2\,E^0_{\alpha\mathbf{k}})^4} - \frac{12\,P_0''(E^0_{\alpha\mathbf{k}})}{(2\,E^0_{\alpha\mathbf{k}})^5} + \frac{60\,P_0'(E^0_{\alpha\mathbf{k}})}{(2\,E^0_{\alpha\mathbf{k}})^6} - \frac{120\,P_0(E^0_{\alpha\mathbf{k}})}{(2\,E^0_{\alpha\mathbf{k}})^7}\right] \tag{7}$$

The Fermi level is measured from the middle of the energy gap. Owing to the symmetry, the integral from the conduction band bottom to the Fermi level $\mu$ in the conduction band is equal to minus integral from the valence band top to the Fermi level $-\mu$ in the valence band. This allows us to write the total Hall conductivity in a following way:

$$\sigma_{xy} = \sigma^0_{xy} - \Delta\sigma_{xy} \tag{8}$$

The term $\sigma^0_{xy}$ is independent of the Fermi level position (the contribution from the fully occupied valence band) while $\Delta\sigma_{xy}$ is Fermi level dependent.

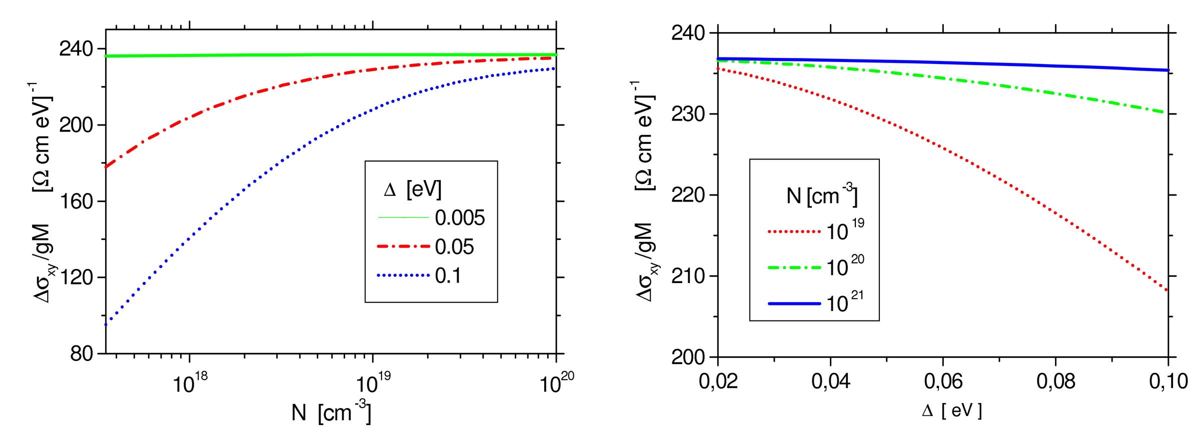


FIGURE 2: The term $\Delta\sigma_{xy}$ as a function of the parameter $\Delta$ and impurity (hole) concentration $N$. The other parameters are as in Fig.1.

Since the valence band in the Dirac model assumed here is not bounded from the bottom, we have to impose some cut-off band edge to calculate $\sigma^0_{xy}$. The corresponding contribution from the whole valence band depends on the assumed band width $W$. It also depends on the parameter $\Delta$ for a particular value of $W$.

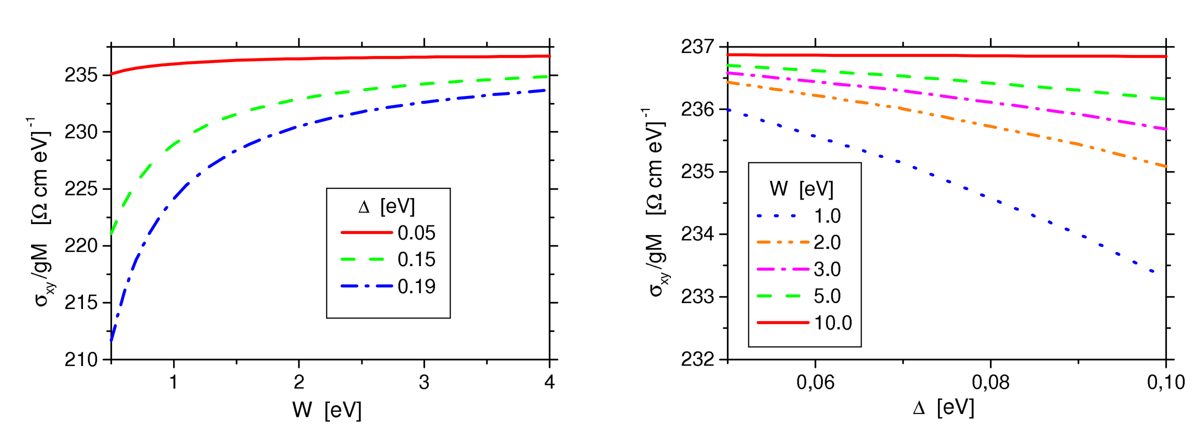


FIGURE 3: Contribution $\sigma^0_{xy}$ to the Hall conductivity as a function of the parameter $\Delta$ and band width $W$. The other parameters are as in Fig.1.

## *Experimental results*

In Fig. 4 we present the temperature dependence of the anomalous Hall constant in two set of ferromagnetic IV-VI compounds (see also Ref. [6]).

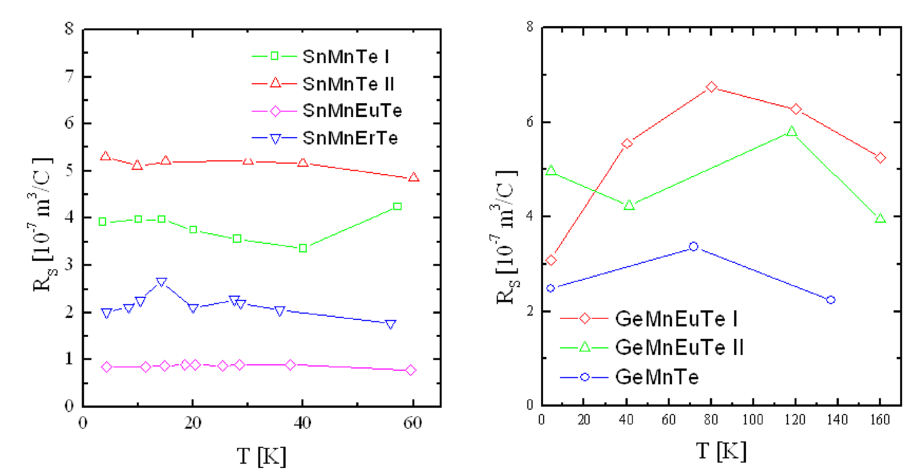


FIGURE 4: The temperature dependence of the anomalous Hall coefficient for semimagnetic semiconductors based on SnTe (The Curie temperature in the range 10-20 K. The content of magnetic constituent (Mn,Er and Eu) does not exceeded 14 at.%.) and GeTe (Curie temperature contained between 80 and 160 K.The content of magnetic constituent reached 38 at.%.).The transport and magnetic measurements (up to 13 and 9 T respectively) were performed for the same set of temperatures.

The anomalous Hall coefficients were determined from the total transverse resistivity and magnetization data by the least square root fit to the equation $\rho_{xy} = R_0B + \mu_0R_SM$, where $B$ is the magnetic field, $R_0$ and $R_S$ are the normal and anomalous Hall coefficients, respectively, and $\mu_0$ is the permeability constant. According to presented results, one can see that within the experimental error the anomalous Hall coefficient does not depend on temperature.

## *Conclusions*

Our considerations do not include impurities.It is well known that impurities can affect strongly the part of AHE related to the the Fermi level but they do not influence the topological contribution. Therefore we think that our findings concerning the AHE in the insulating regime (with the Fermi level in the gap) should not depend on impurities. Besides, we can also expect the leading role of the topological mechanism of AHE in case of doped semiconductors due to the large SO interaction in the crystal potential whereas the impurity potential can be rather small.

As follows from our calculations, the off-diagonal conductivity does not vanish when the Fermi level is located within the energy gap. We calculated it numerically as a certain value $\sigma^0_{xy}$, which does not depend on any band parameters but the magnitude of spin splitting in the valence band. In view of the strong spin-orbit interaction, it is not possible to separate contributions from the spin up and down states. As a result, both magnetically splitted bands contribute to $\sigma^0_{xy}$.

Unfortunately, it was rather difficult to compare theoretical results with experiment, because the experimental curves are measured on samples with different contents of Mn. However, by extracting the dependence of the AHE prefactor on the carrier density, we found that these dependences are qualitatively similar. From a qualitative comparison of the experimental and theoretical results we come to the conclusion that the topological contribution to the AHE is not negligible, although not sufficient to account for the experimental observations.

## References

[1] A. Crépieux and P. Bruno, Phys. Rev. B **64**, 014416 (2001).

[2] H. Ohno and F. Matsukura, Sol. State Commun. **117**, 179 (2001).

[3] V. K. Dugaev, A. Crépieux, and P. Bruno, Phys. Rev. B **64**, 104411 (2001).

[4] G. Nimtz and B. Schlicht, in *Narrow Gap Semiconductors.* Springer Tracts in Modern Physics, vol. 98 (Springer, Berlin, 1983), p. 1.

[5] T. Story, R. R. Gałązka, R. B. Frankel, and P. A. Wolf, Phys. Rev. Lett. **56**, 777 (1986).

[6] B. Brodowska, W. Dobrowolski, M. Arciszewska, E. I. Slynko, and V. K. Dugaev, J. Alloys and Compounds **423**, 205 (2006).