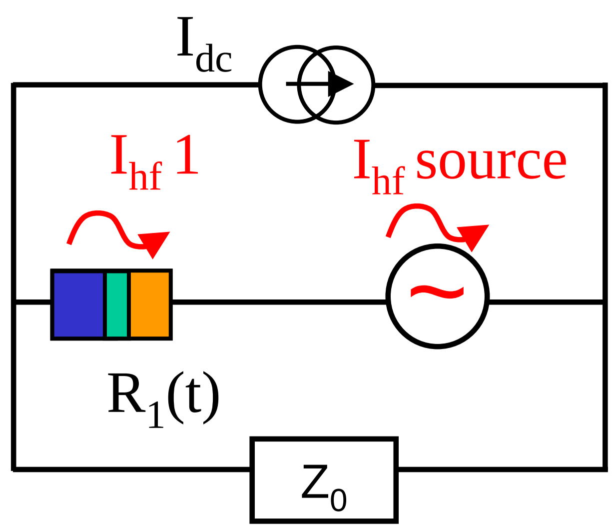
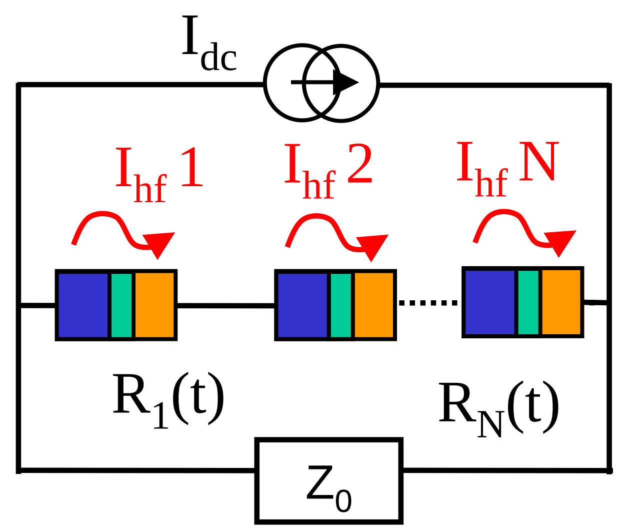
Phase locking of a Spin Transfer Nano-Oscillator to an external microwave current :





J. Grollier[1], B. Georges[1], M. Darques[1], V. Cros[1], G. Faini[2], C. Deranlot[1], B. Marcilhac[1], A. Fert[1]

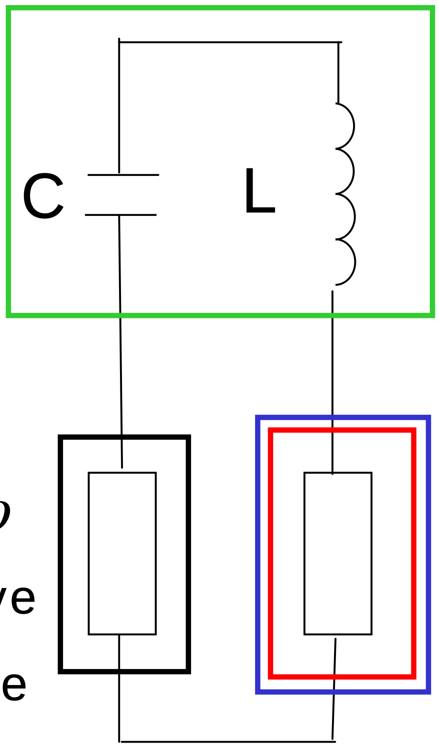
1 Unité Mixte de Physique CNRS/Thales et Université Paris Sud, Palaiseau, France

2 LPN CNRS, Marcoussis, France

LLG equation

$$\frac{d\vec{m}}{dt} = -\gamma_0\, \vec{m} \times \vec{H}_{eff} + \alpha\, \vec{m} \times \frac{d\vec{m}}{dt} + \gamma_0\, J\, \vec{m} \times (\vec{m} \times \vec{M})$$

$$\omega = \gamma H_{eff} = \frac{1}{\sqrt{LC}}$$



$R_0 \propto \alpha_G \omega$

Dissipative resistance

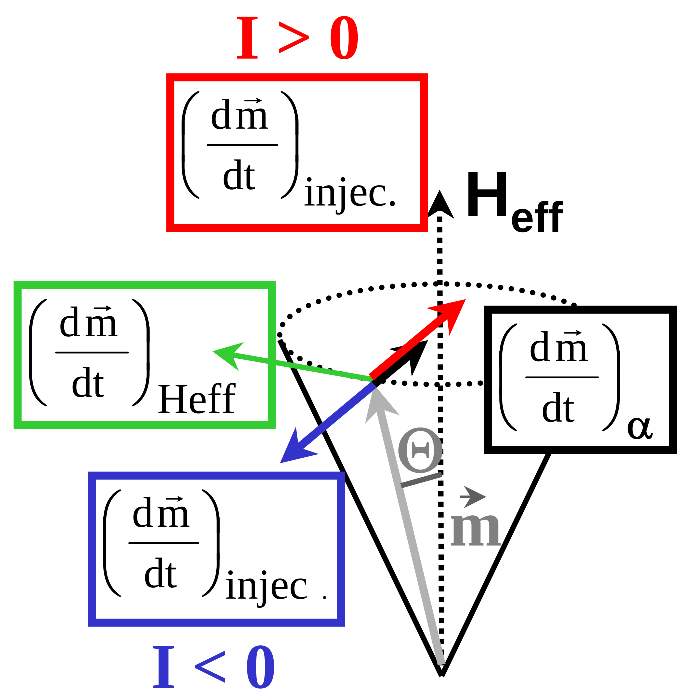
$-R_S \propto \gamma_0 J$

Negative resistance

Precession if :

$R_0 - R_s = 0$

Spin torque compensates the damping



A. Slavin model *IEEE* **41**, 4 (2005)

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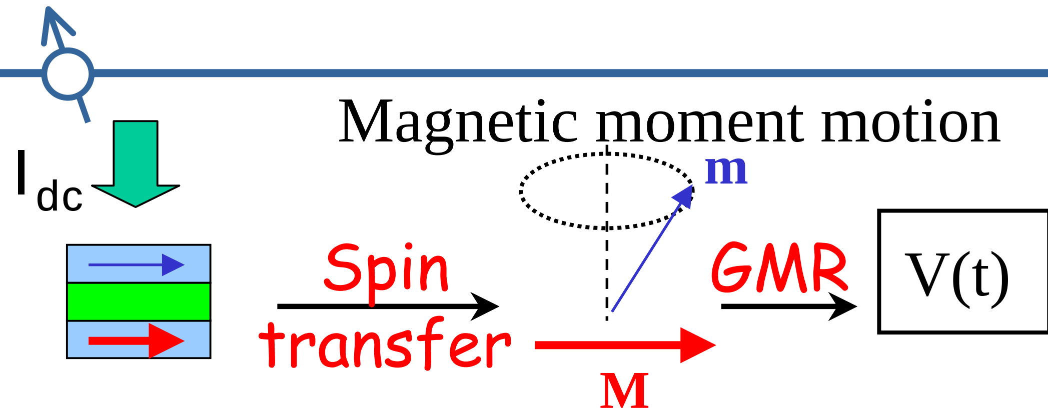
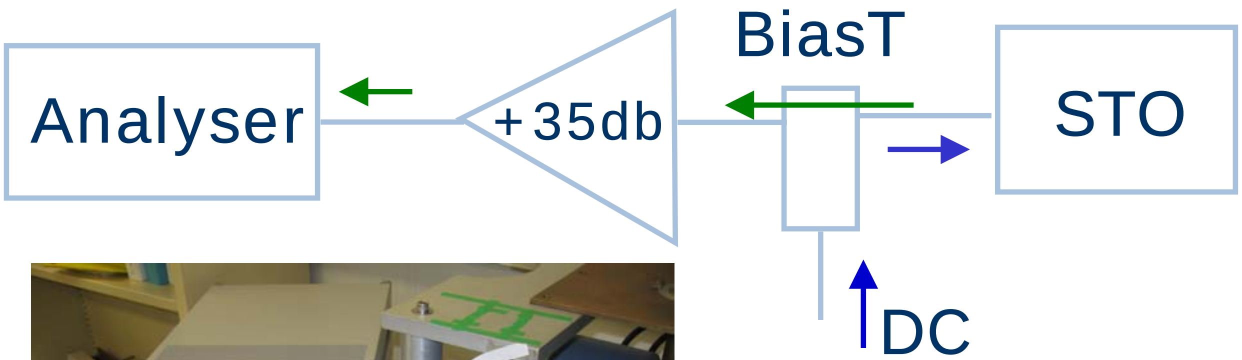
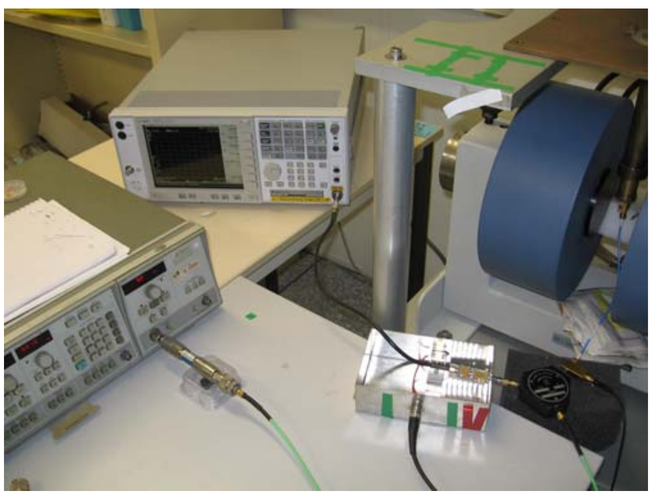
THALES

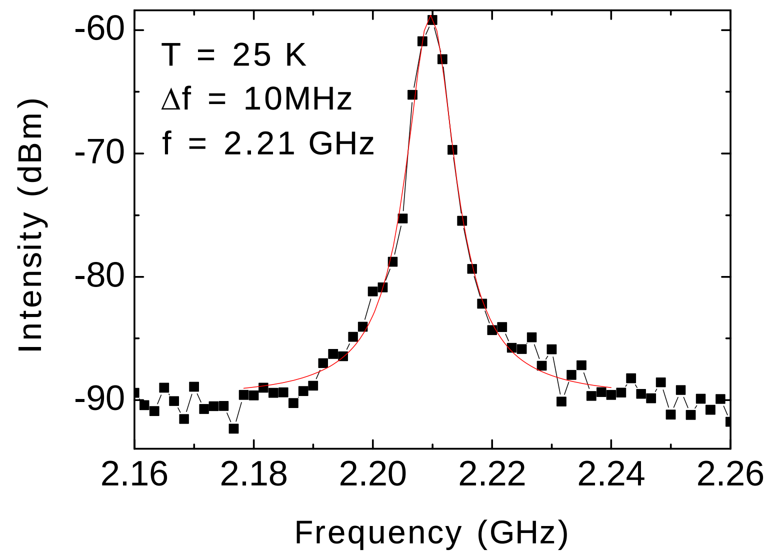


$$V(t) = \Delta R_{\max} I_{dc} \vec{m}(t) \cdot \vec{M}$$



$$P \approx \frac{\Delta R_{osc}^2 \; I_{dc}^2}{Z_{LOAD}}$$





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THALES

- *non-linear ($I_{dc} \rightarrow V_{ac}$)*
- *agile (I_{dc}, H)*
- *direct emission up to 40 GHz*
- *high Q factor*
- *sub-micron size*


applications :
Telecommunications
RADAR

Problem : very weak emitted power

P_{out} ~ **- 60 dBm** Too low for applications

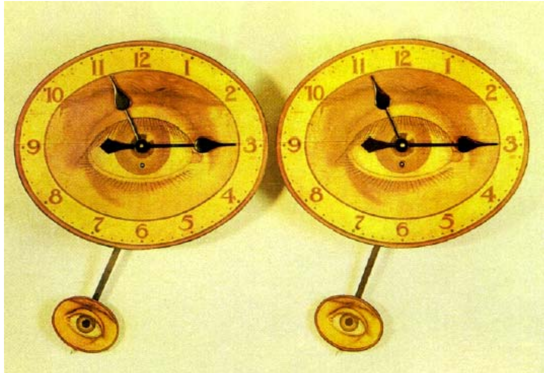
Solution : synchronization

Coherent emission in frequency and phase of many oscillators

Synchronization : the oldest non-linear effect ever studied!!

Fireflies

Huygens Pendulum clocks (1665)



Applauding audience

- Other exemples : criquets, applauding audience, pace maker cells, intestine celluls, circadian rythm, Josephson junctions etc.

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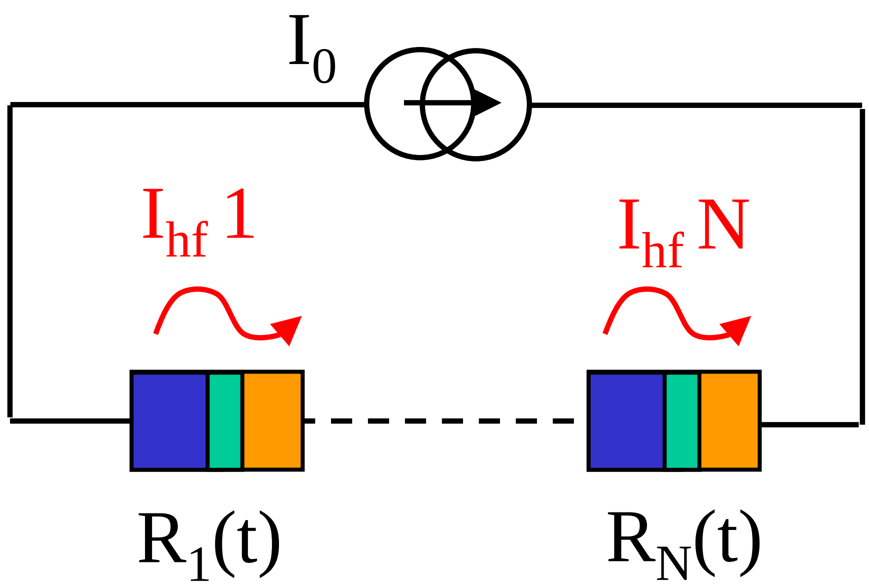
THALES

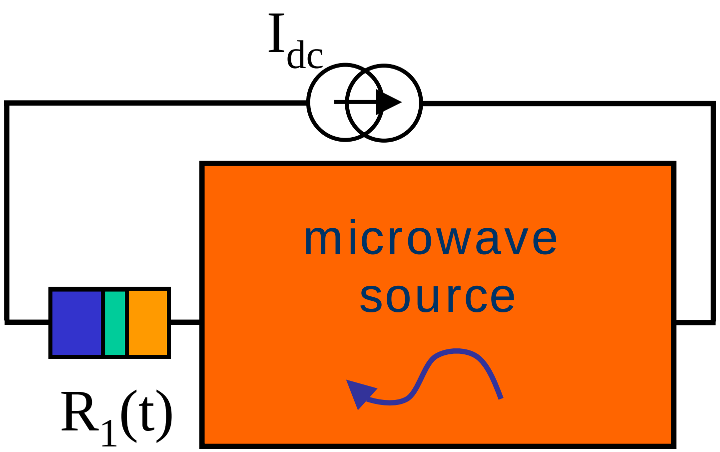
Non local coupling:

self-emitted

microwave currents



Electrically connected oscillators



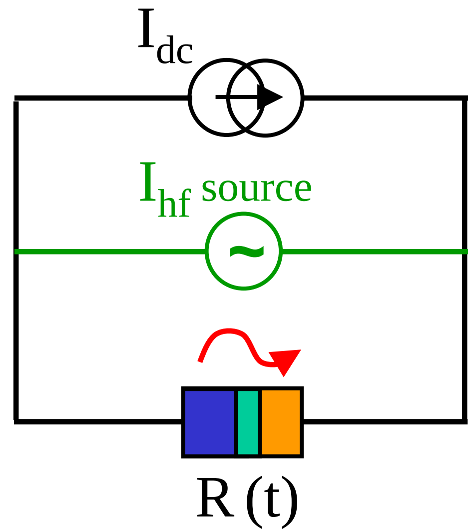
Current through ONE oscillator

$$I_{tot} = I_{dc} + \sum_{i=1}^{N} I_{hf}(i)$$

f_o

$\Delta f,\ \Delta\varphi$

Injection of a microwave signal



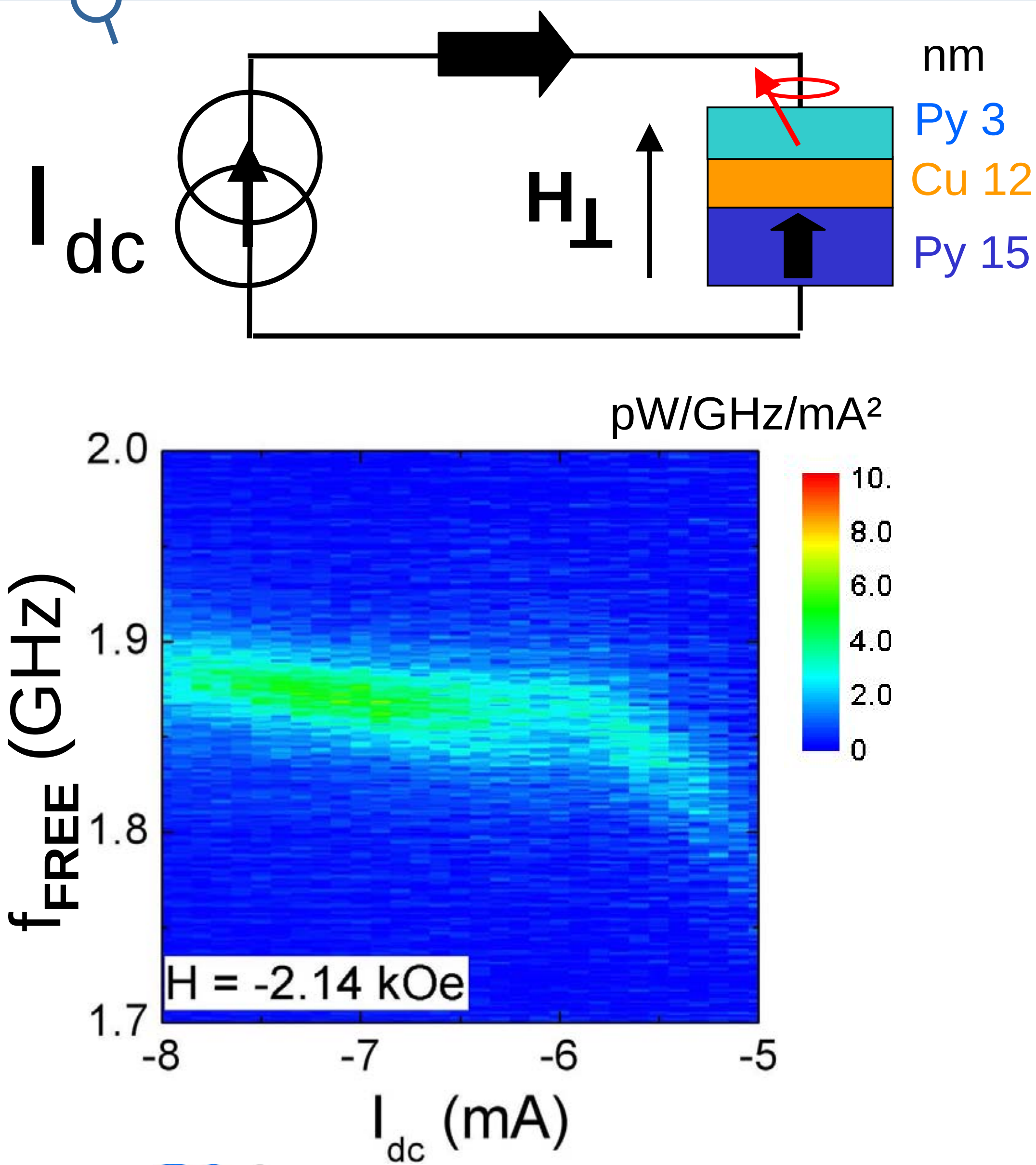
Current through THE oscillator

$$I_{tot} = I_{dc} + I_{hf}\ source$$

f_o

$\Delta f,\ \Delta\varphi$

Features of a single STNO

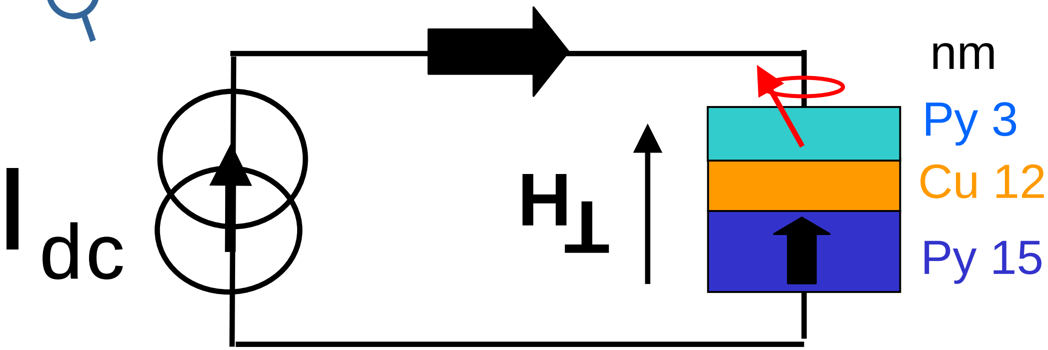


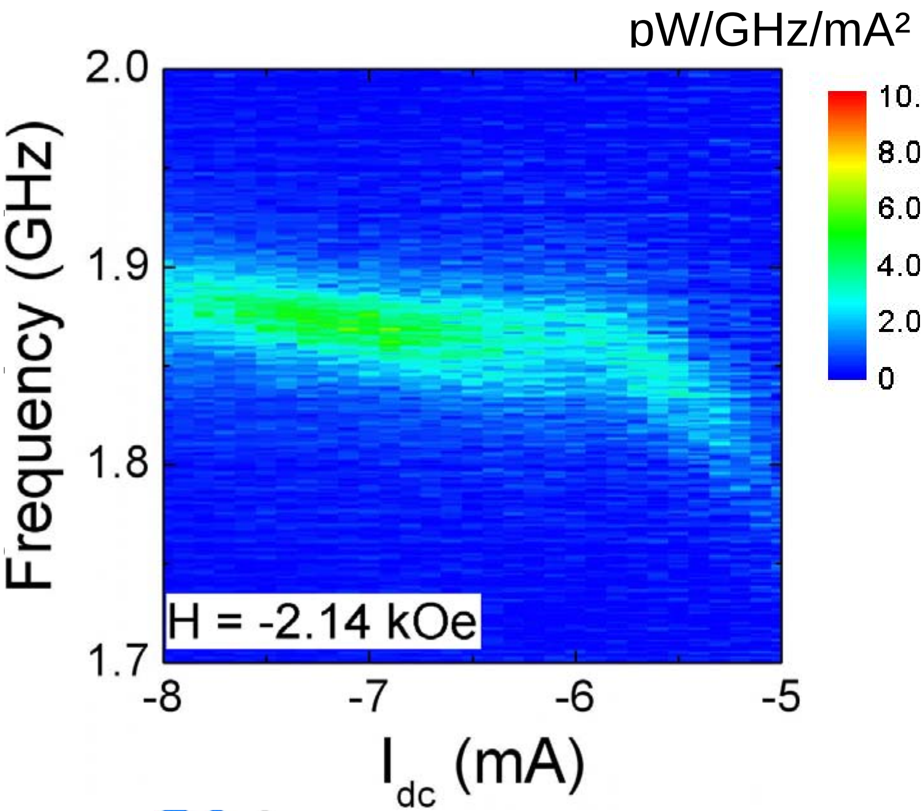
microwave characteristics of the emission of an STNO:

- frequency
- linewidth
- power
- agility in current (df_{FREE}/dI_{dc})

change with the experimental conditions (H and I_{dc})

Features of a single STNO





microwave characteristics of the emission of an STNO:

- frequency
- linewidth
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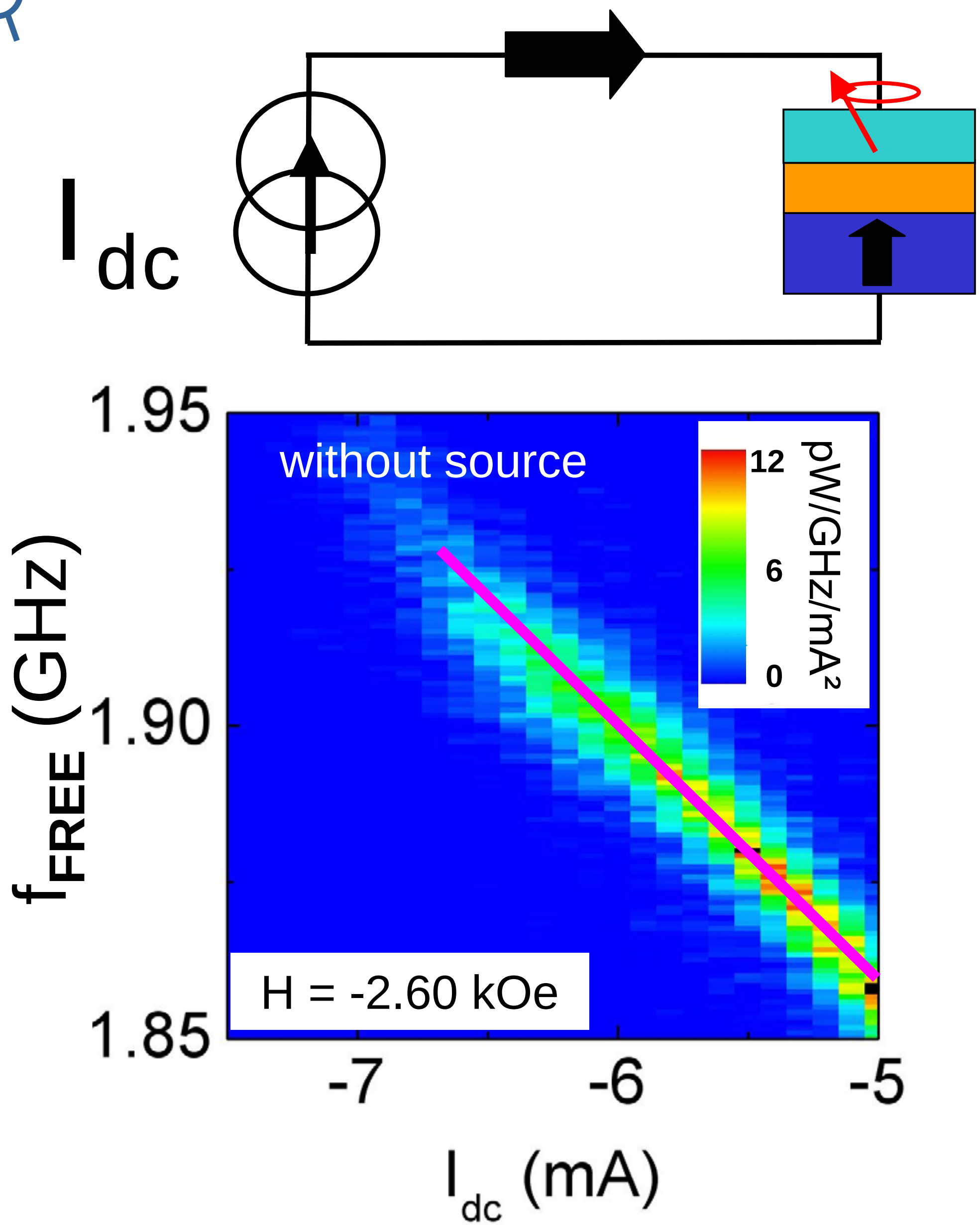
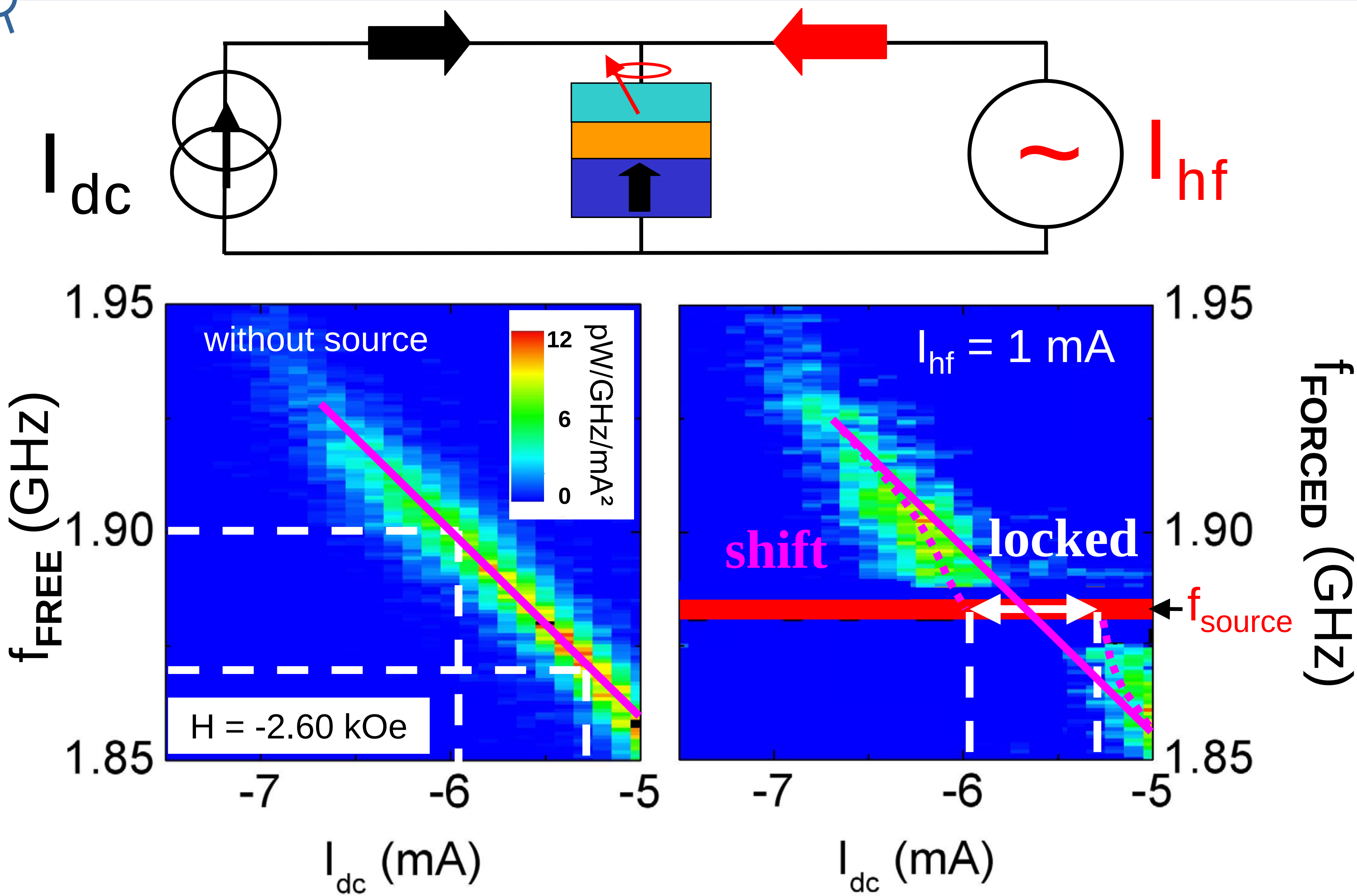
change with the experimental conditions (H and I_{dc})

Phase locking to an external source

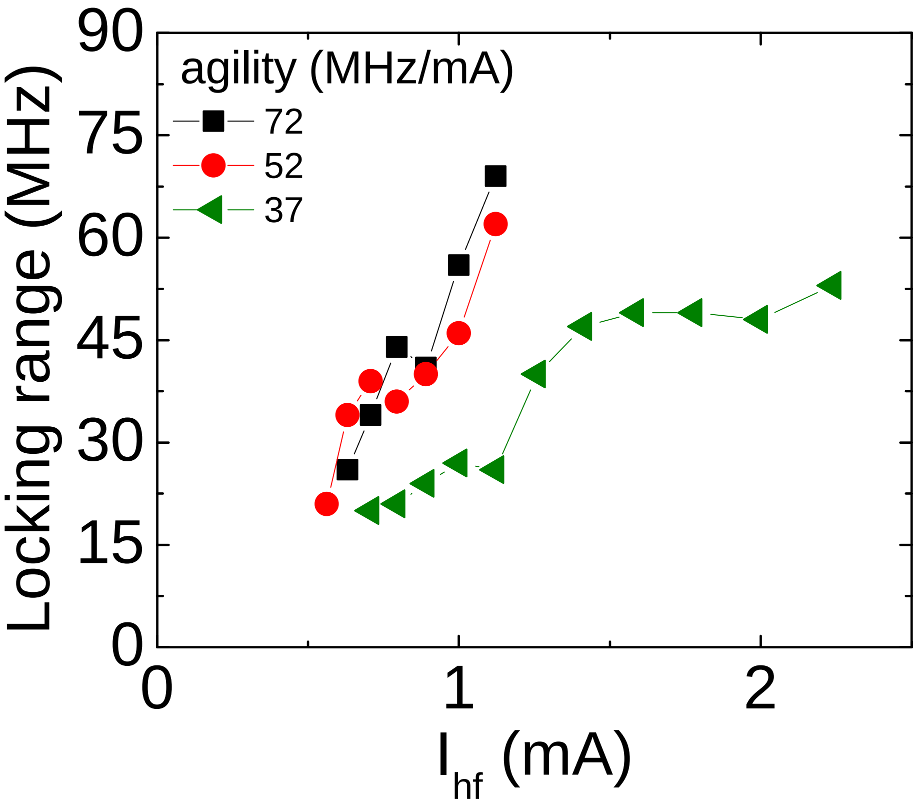


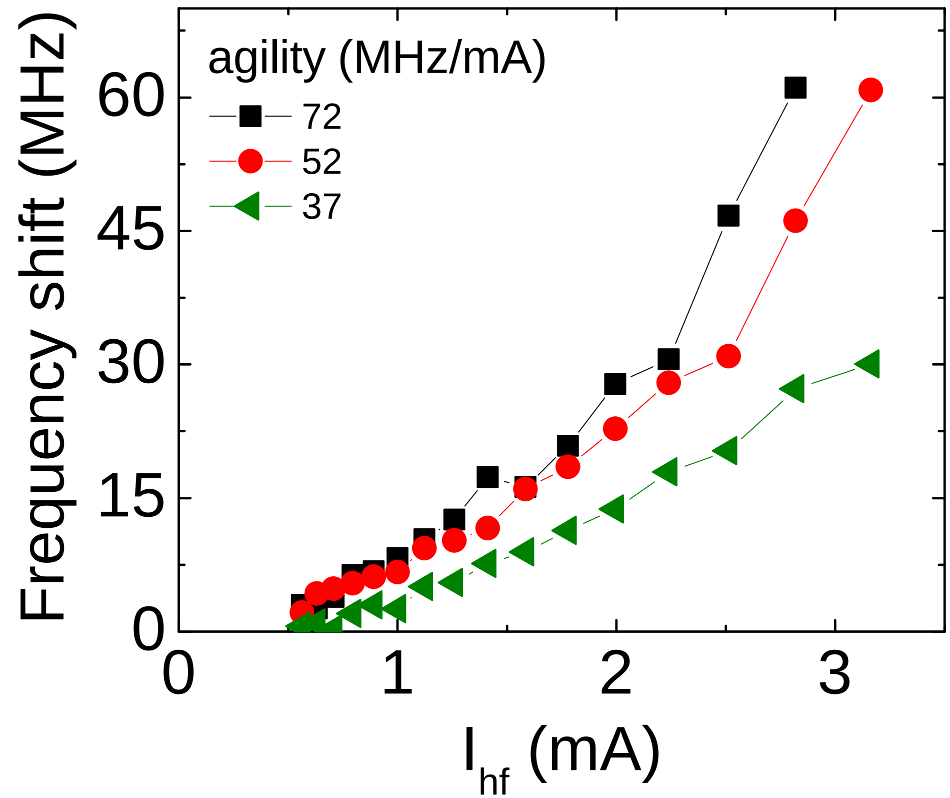
Phase locking to an external source



Locking range and frequency shift

(H, I_{dc}) chosen so that only the agility in current changes

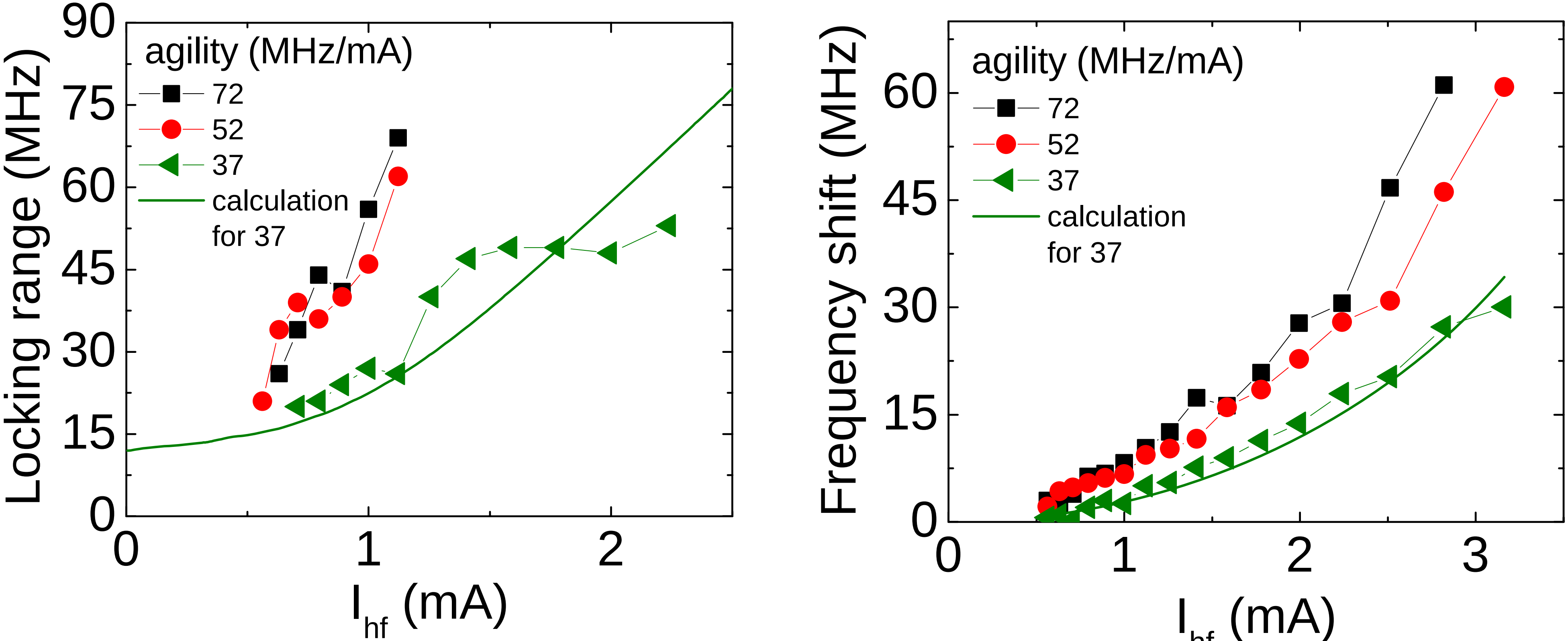




the microwave characteristics of the STNO
determine the coupling strength to the external microwave current

Locking range and frequency shift

(H, I_{dc}) chosen so that only the agility in current changes



the microwave characteristics of the STNO
determine the coupling strength to the external microwave current

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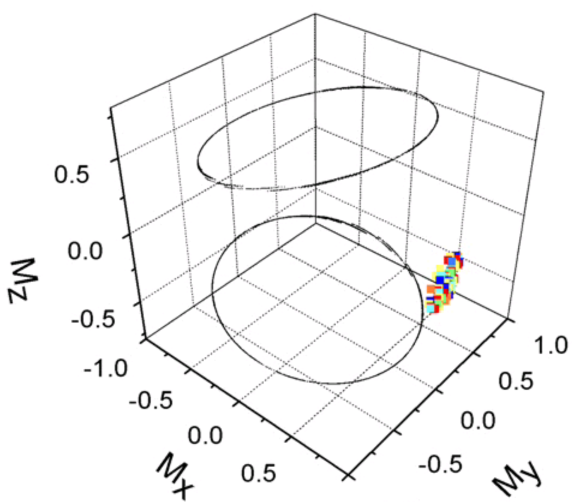
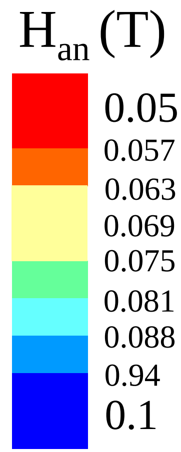
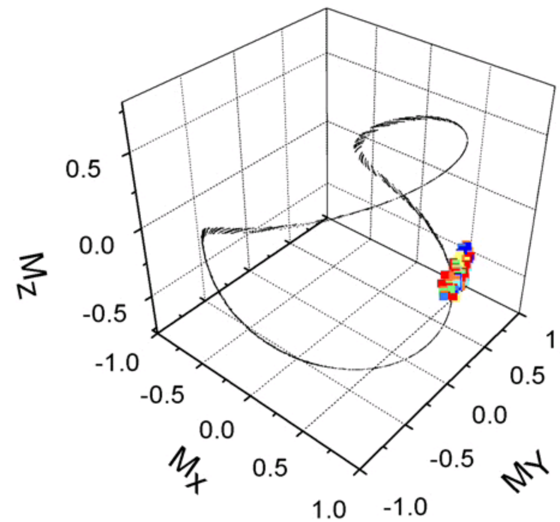
THALES

Synchronization / Phase dynamics (*100 oscillators*)



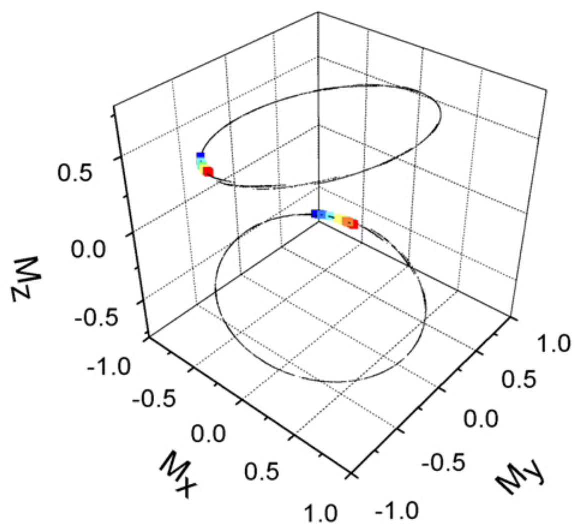
$J = 0.035$ T
$\tau = 5$ ps
$A_{GMR} = 0.03$



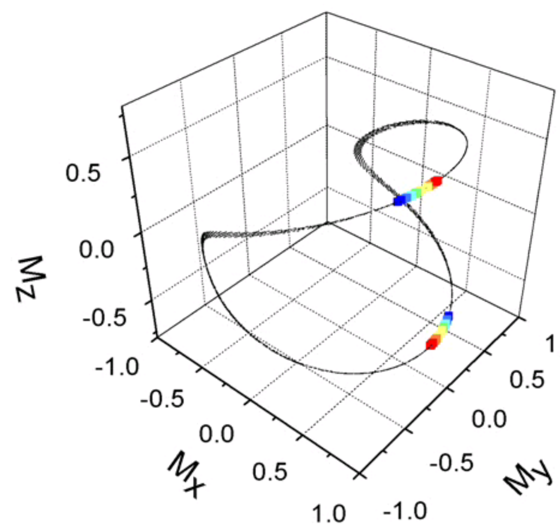


$J = 0.01$ T
$\tau = 300$ ps
$A_{GMR} = 0.4$

Macrospin simulations



SYNC : adjustment of the PHASE



equation of the magnetization motion

$$\frac{db}{dt} = -i\left(\omega_{FMR} + Nb^2\right)b - \Gamma\left(1 - Qb^2\right)b + \sigma I_{dc}\left(1 - b^2\right)b$$

rotation damping spin transfer

$$b = ce^{i\varphi}$$

Spin-wave : amplitude c and phase φ

A. Slavin *et al. IEEE* **41**, 4 (2005)

uniformly rotating phase

A. Pikovsky, *Synchronization, A universal concept in nonlinear sciences*, Cambridge Nonlinear Science Series 12 (2001)

$$\phi = \varphi + \frac{N}{\sigma I_{dc} + \Gamma Q}\ln(c) + \phi_0$$

valid even when slightly perturbed

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$$\frac{db}{dt} = -i\left(\omega_{FMR} + Nb^2\right)b - \Gamma\left(1 - Qb^2\right)b + \sigma I_{dc}\left(1 - b^2\right)b + \frac{\sigma I_{hf}}{2\sqrt{2}}\tan(\gamma)e^{-i\varpi_s t}$$

external source term

phase dynamics:

$\Delta\phi = \phi - \phi_{source}$

$$\frac{d(\Delta\phi)}{dt} = 2\pi\left(f_{FREE} - f_{source}\right) + \varepsilon\sin(\Delta\phi) + \xi(t)$$

detuning — coupling strength — noise w^2

Adler equation

R. Adler, *IEEE*, 61, **10** (1973)

coupling strength

$$\varepsilon = \frac{I_{hf}}{2\sqrt{2}}\sigma\tan(\gamma)\sqrt{\frac{I_{dc}}{I_{dc} - I_{th}}}\sqrt{1 + \left(\frac{2\pi}{\sigma}\frac{I_{dc}}{I_{th}}\frac{\partial f_{FREE}}{\partial I_{dc}}\right)^2}$$

A. Slavin *et al.*, Phys. Rev. B, **72**, 092407 (2005)

Phase locking to an external source : principle

General equation of the phase dynamics of a forced oscillator:

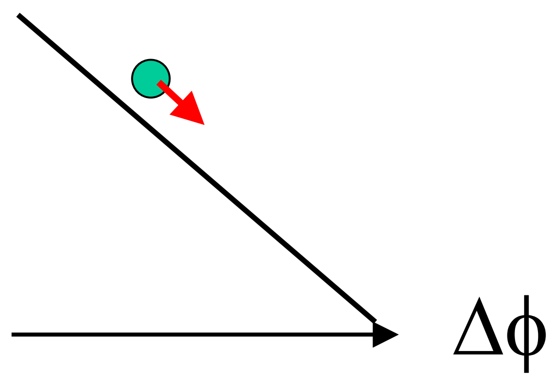
R. Adler, *IEEE*, 61, **10** (1973)

$$\frac{d(\Delta\Phi)}{dt} = \left(\omega_{free} - \omega_{source}\right)$$

detuning

- if $\varepsilon = 0$ $\Delta\Phi = (\omega_{free} - \omega_{source})\, t + \Phi_0$

the STNO and the source evolve independantly



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THALES

General equation of the phase dynamics of a forced oscillator:

R. Adler, *IEEE*, 61, **10** (1973)

$$\frac{d(\Delta\Phi)}{dt} = (\omega_{free} - \omega_{source}) + \varepsilon \sin(\Delta\Phi)$$

detuning

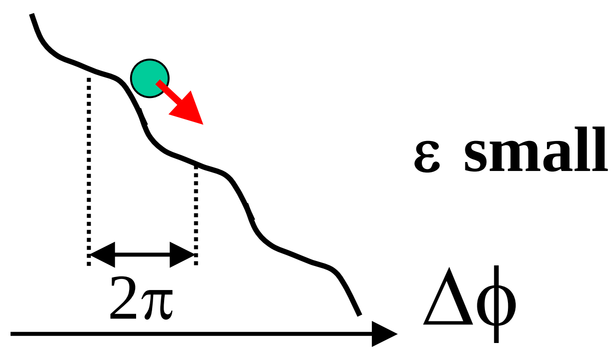
coupling strength

- if $\varepsilon \neq 0$

$\Delta\Phi$ is alternatively increased and decreased

Indeed : $I_{tot} = I_{dc} + I_{hf} \sin(\omega_{source} t)$

ω_{free} is alternatively increased and decreased



General equation of the phase dynamics of a forced oscillator:

R. Adler, *IEEE*, 61, **10** (1973)

$$\frac{d(\Delta\Phi)}{dt} = (\omega_{free} - \omega_{source}) + \varepsilon \sin(\Delta\Phi)$$

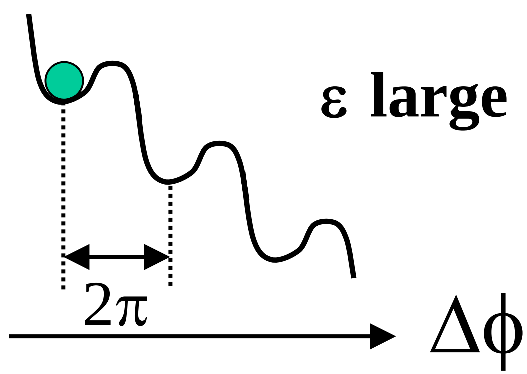
detuning

coupling strength

- if ε > detuning

there is a solution with constant ΔΦ

➡ locking for large couplings



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Phase locking to an external source : principle

General equation of the phase dynamics of a forced oscillator:

R. Adler, *IEEE*, 61, **10** (1973)

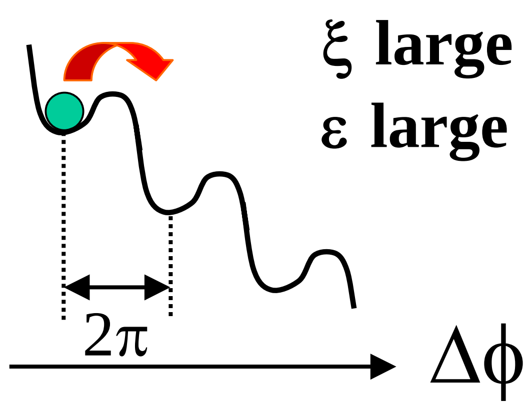
$$\frac{d(\Delta\Phi)}{dt} = \left(\omega_{free} - \omega_{source}\right) + \varepsilon \sin(\Delta\Phi) + \xi(t)$$

detuning

coupling strength

noise σ^2

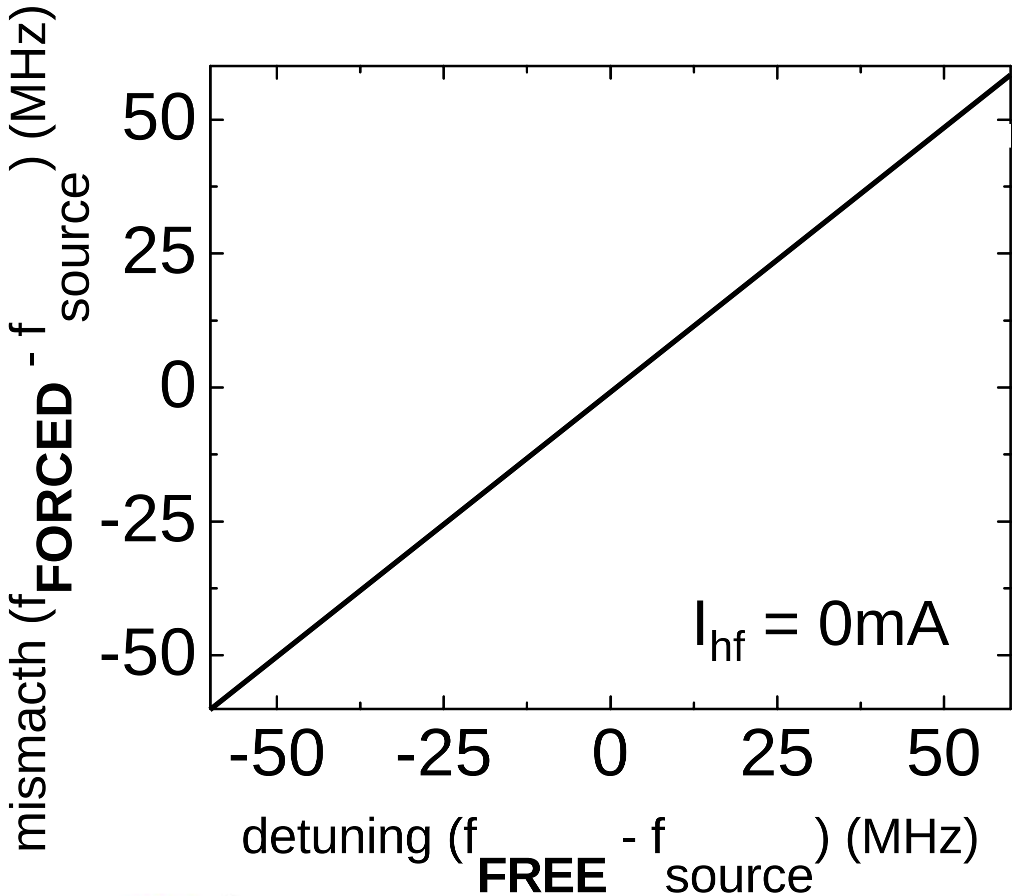
- the noise ξ accounts for frequency fluctuations (linewidth)



Influence of phase noise: experiment vs model

mismatch = $f_{FORCED} - f_{source}$

detuning = $f_{FREE} - f_{source}$



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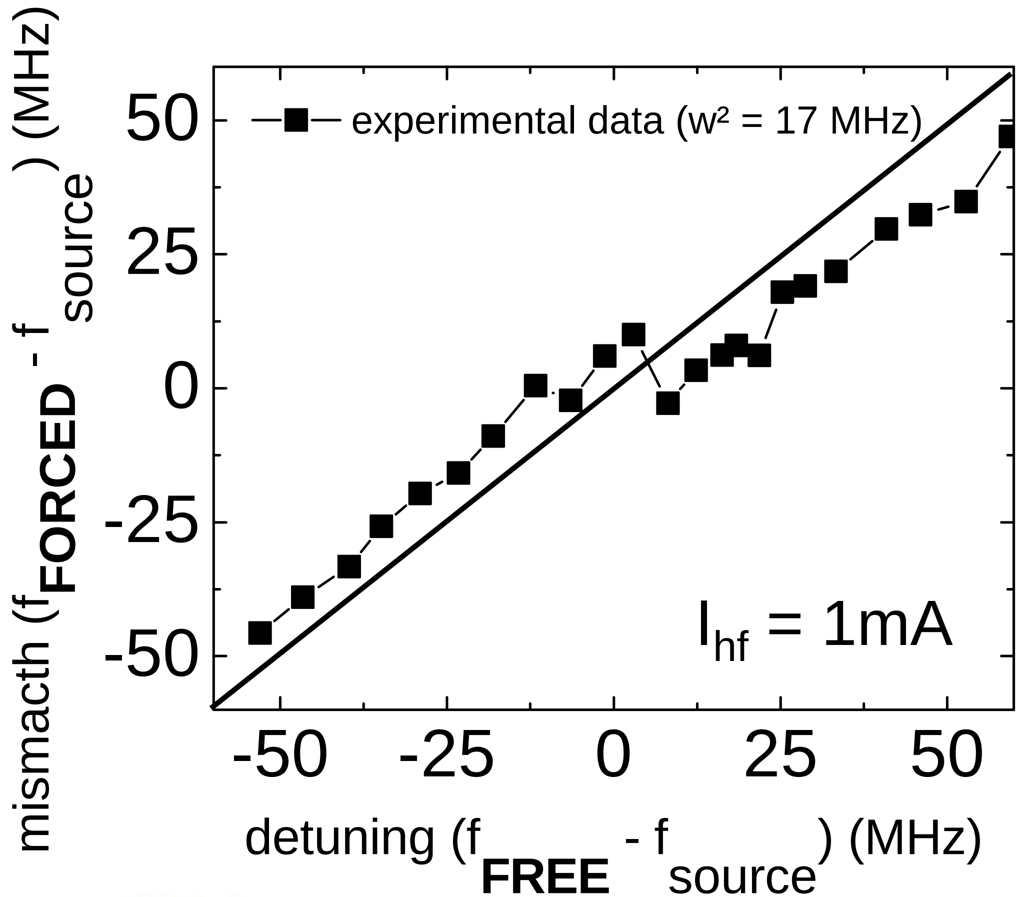
Influence of phase noise: experiment vs model

$$\frac{d(\Delta\phi)}{dt} = 2\pi\left(f_{FREE} - f_{source}\right) + \varepsilon \sin(\Delta\phi) + \xi(t)$$



calculation of the mismatch

depends on:

- detuning ($f_{FREE} - f_{source}$)
- noise w^2 (linewidth)
- coupling strength ε (free parameter)

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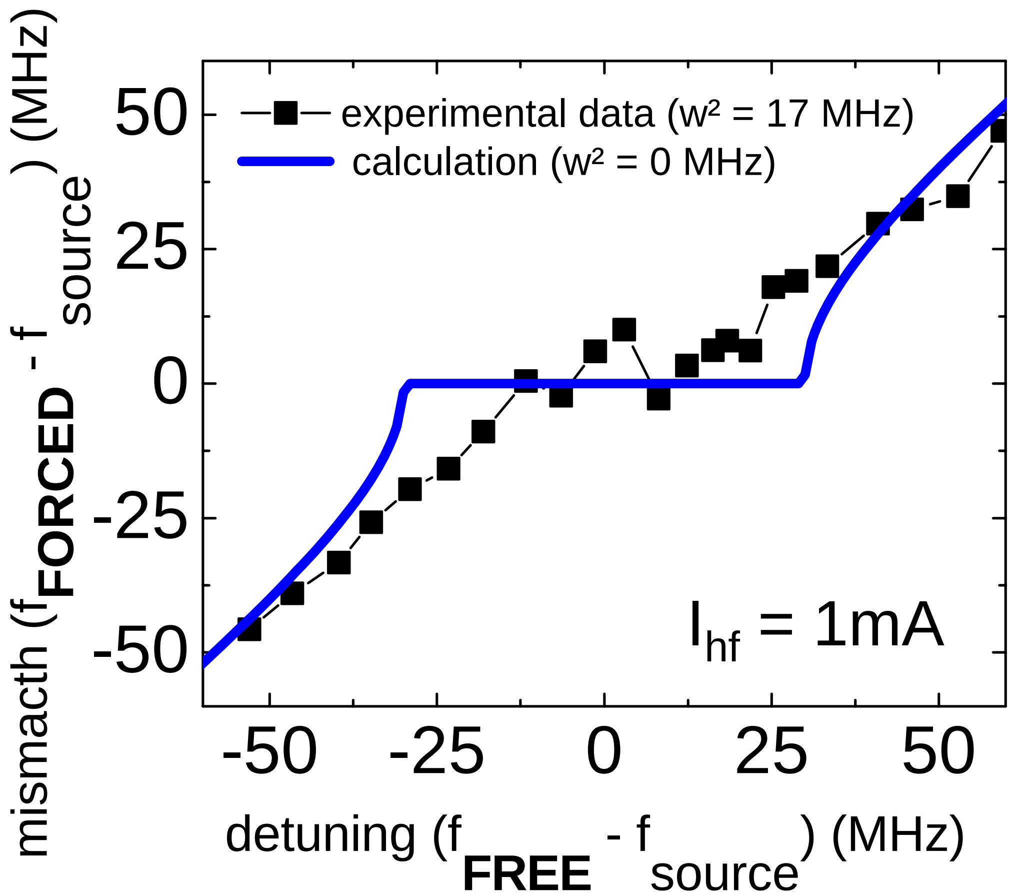
Influence of phase noise: experiment vs model

$$\frac{d(\Delta\phi)}{dt} = 2\pi\left(f_{FREE} - f_{source}\right) + \varepsilon \sin(\Delta\phi) + \xi(t)$$



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Influence of phase noise: experiment vs model

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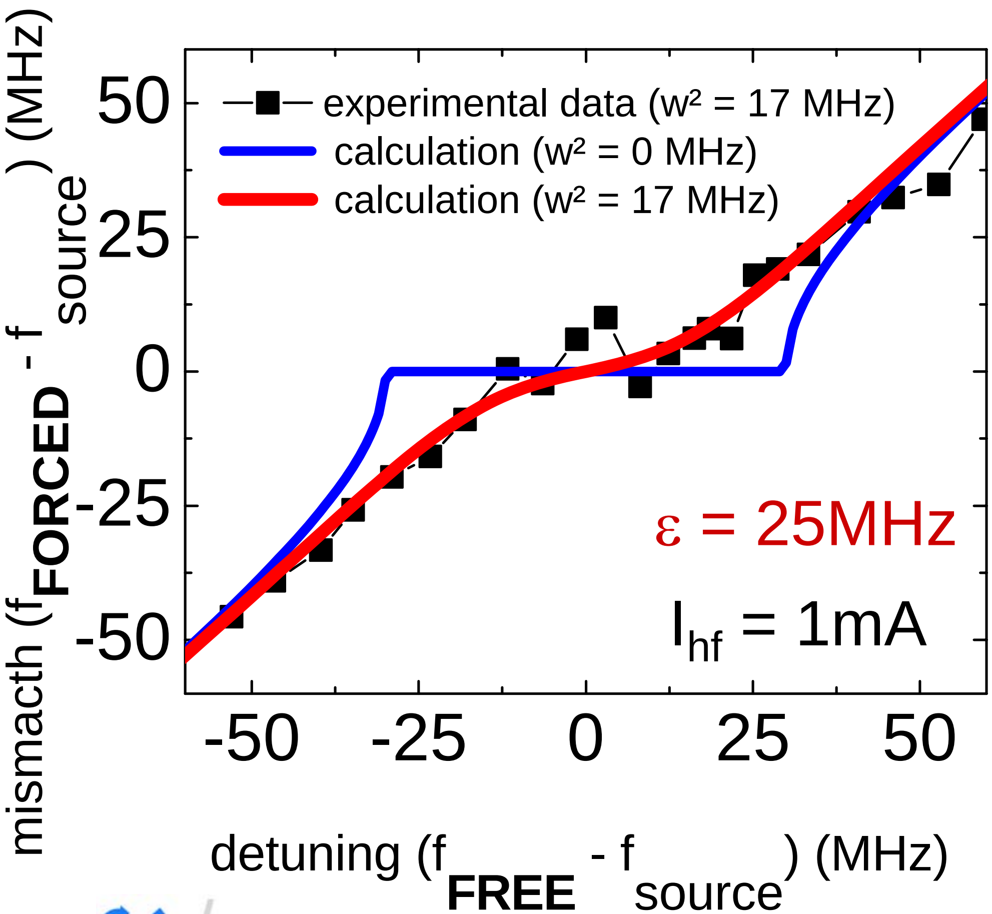

calculation of the mismatch

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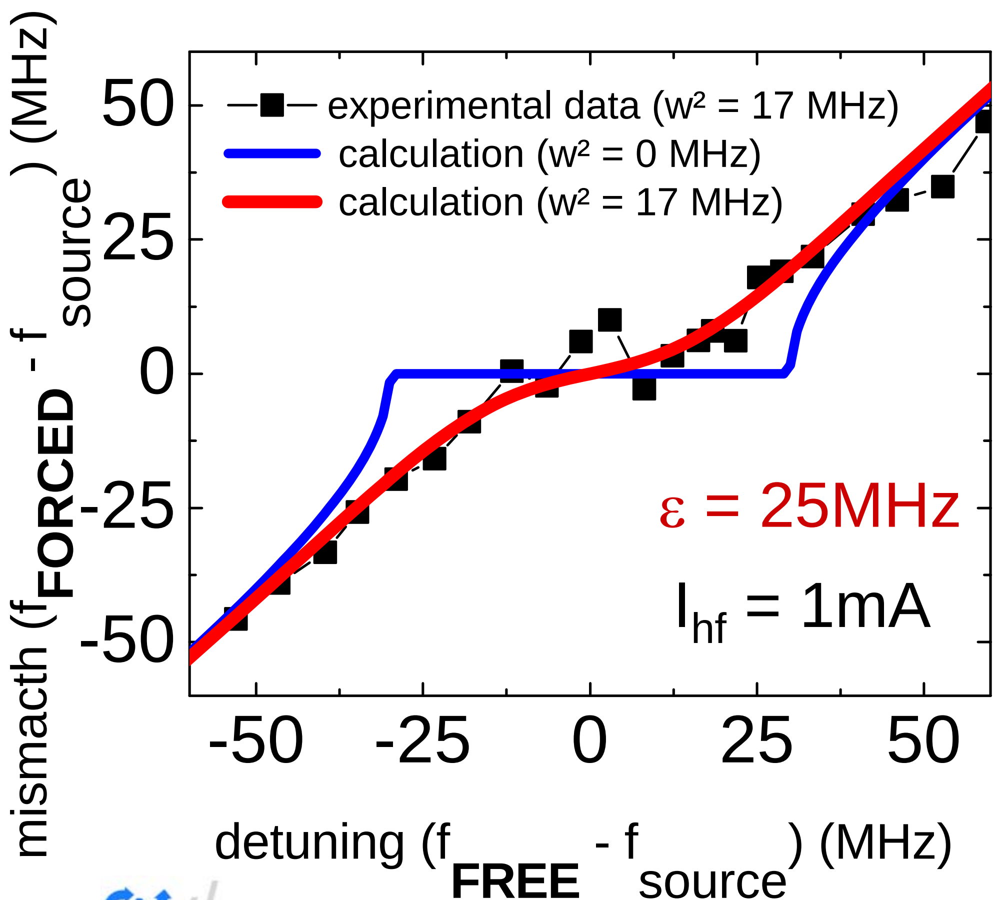

calculation of the mismatch

depends on:

- detuning ($f_{FREE} - f_{source}$)
- noise w^2 (linewidth)
- coupling strength ε (free parameter)

➢ forced STNOs can be described with the Adler model of phase locking

➢ critical influence of phase noise

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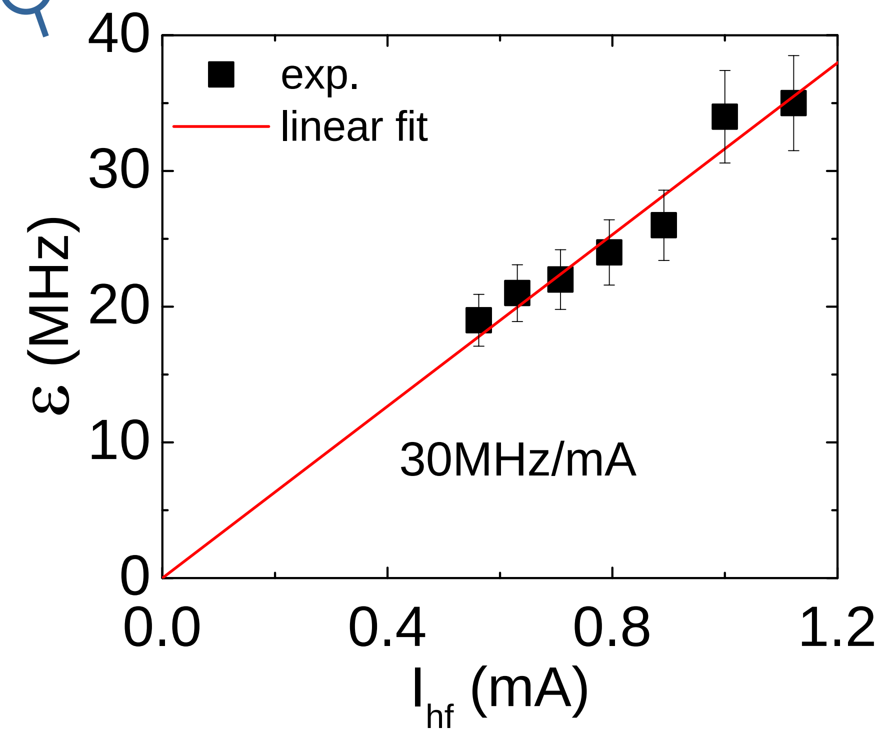
THALES

Experimental test of the coupling calculation



$$\varepsilon = \boxed{I_{hf}} \boxed{\frac{\sigma \tan(\gamma)}{2\sqrt{2}} \sqrt{\frac{I_{dc}}{I_{dc}-I_{th}}} \sqrt{1+\left(\frac{2\pi}{\sigma}\frac{I_{dc}}{I_{th}}\frac{\partial f_{FREE}}{\partial I_{dc}}\right)^2}}$$

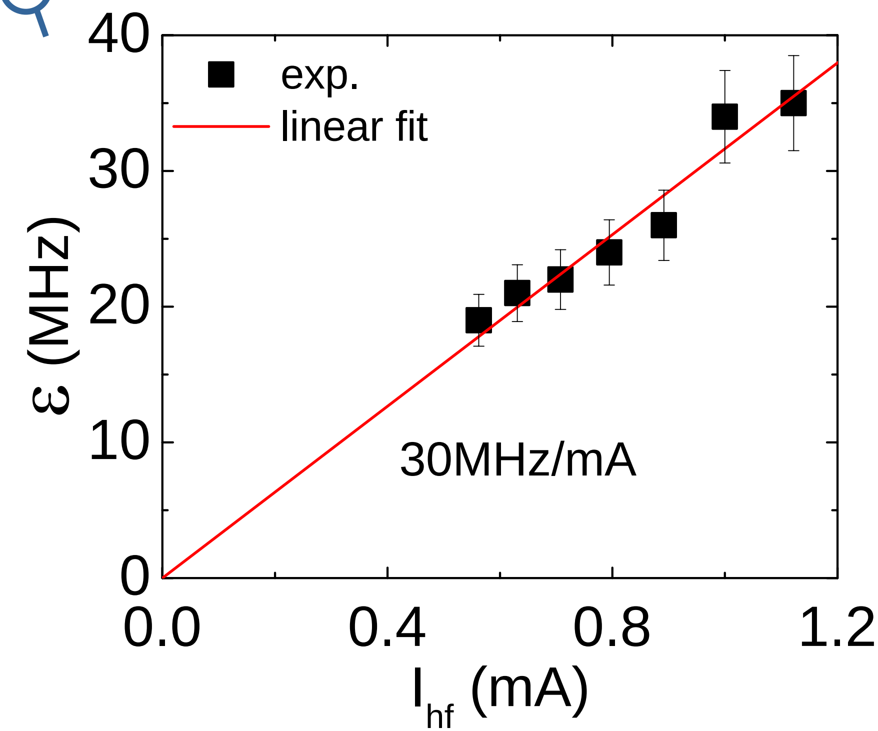
varying — slope

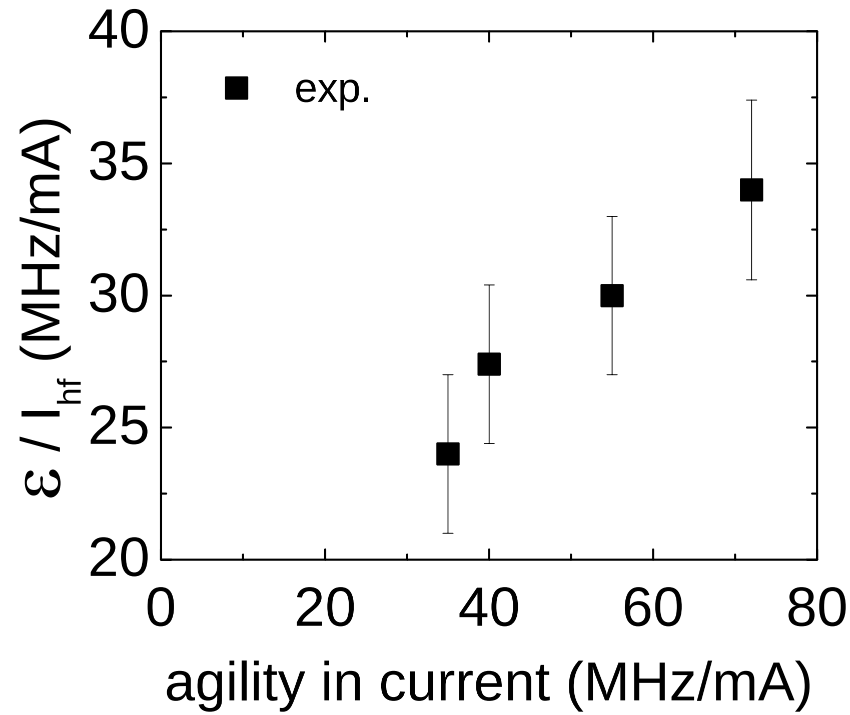
⇒

with σ= 825 MHz/mA

=> γ = 2.75°

Experimental test of the coupling calculation

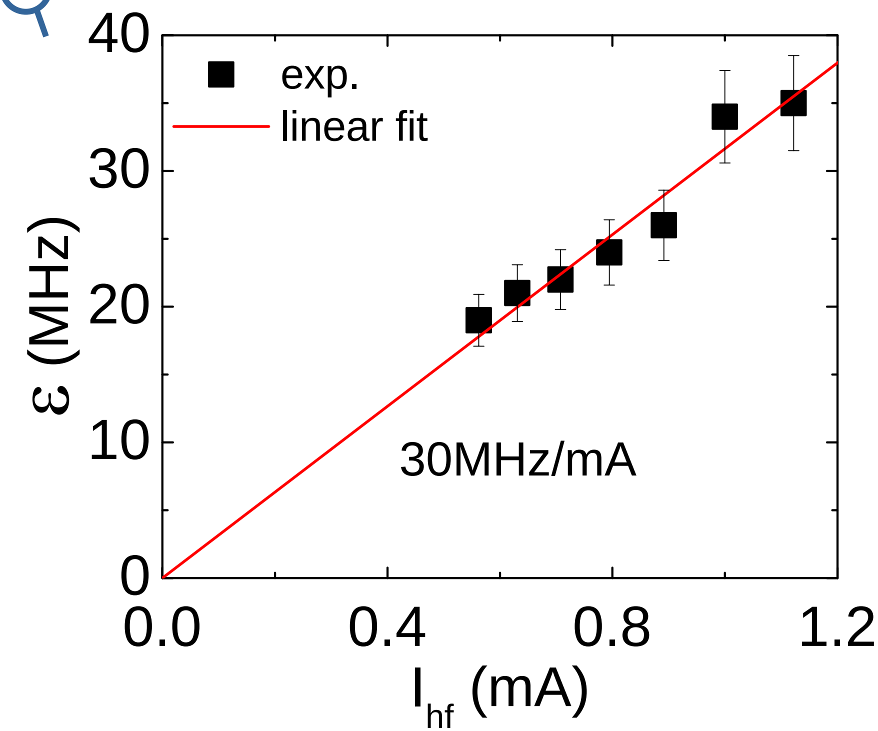


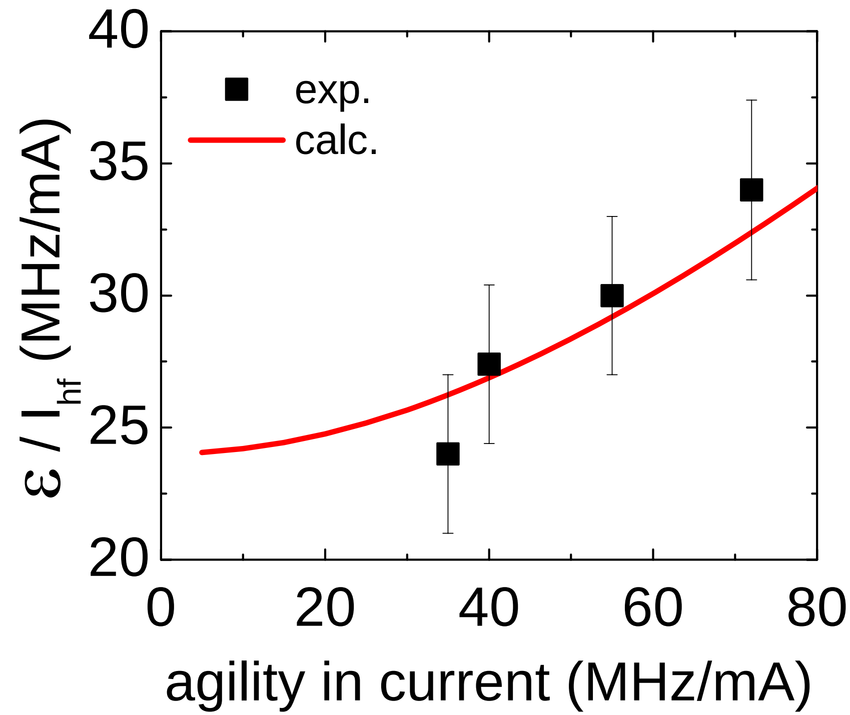


$$\varepsilon = I_{hf} \frac{\sigma \tan(\gamma)}{2\sqrt{2}} \sqrt{\frac{I_{dc}}{I_{dc} - I_{th}}} \sqrt{1 + \left(\frac{2\pi}{\sigma} \frac{I_{dc}}{I_{th}} \frac{\partial f_{FREE}}{\partial I_{dc}} \right)^2}$$

constant varying

Experimental test of the coupling calculation





with σ= 825 MHz/mA (s=0.31)

=> γ = 2.75°

$$\varepsilon = I_{hf} \frac{\sigma \tan(\gamma)}{2\sqrt{2}} \sqrt{\frac{I_{dc}}{I_{dc} - I_{th}}} \sqrt{1 + \left(\frac{2\pi}{\sigma} \frac{I_{dc}}{I_{th}} \frac{\partial f_{FREE}}{\partial I_{dc}} \right)^2}$$

constant | varying

high agility enhances the coupling

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Phase locking to an external source : conclusion

qualitative and quantitative understanding of injection locking experiments :

- description with Adler model of forced oscillators
- expression of the coupling strength

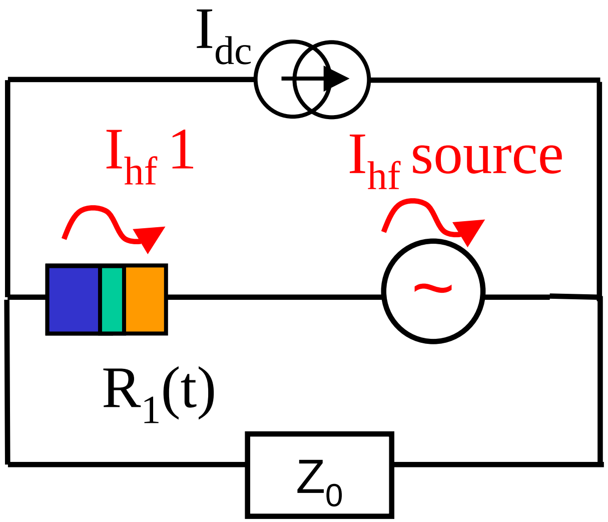
main trends :

- ✓ bad influence of the noise on phase locking
- ✓ high agility enhances the coupling

B. Georges *et al.*, PRL **101,** 017201 (2008)

From Adler equation to the Kuramoto model

forced STNO

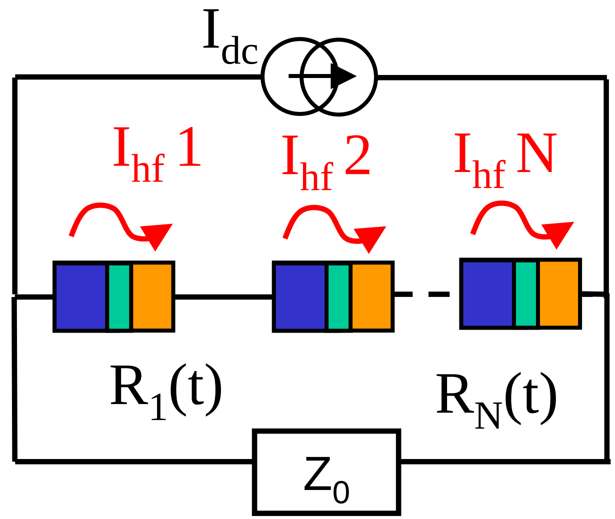


$$\frac{d(\Delta\phi)}{dt} = 2\pi\left(f_{FREE} - f_{source}\right) + \varepsilon \sin(\Delta\phi) + \xi(t)$$

Adler equation

$$\frac{\varepsilon}{I_{hf}} = \frac{\sigma\tan(\gamma)}{2\sqrt{2}}\sqrt{\frac{I_{dc}}{I_{dc}-I_{th}}}\sqrt{1+\left(\frac{2\pi}{\sigma}\frac{I_{dc}}{I_{th}}\frac{\partial f_{FREE}}{\partial I_{dc}}\right)^2}$$

N STNOs connected in series



$$\frac{d\Delta\phi_n}{dt} = 2\pi\Delta f_n + \frac{K}{N}\sum_n \cos\Delta\phi_n + \xi_n(t)$$

Kuramoto model

$$K = \left(\frac{\varepsilon}{I_{hf}}\right).\frac{N}{Z_0 + NR}\Delta R_{osc}\, I_{dc}$$

Synchronization threshold $K_c = 2(w^2 + D)$

$$K = \left(\frac{\varepsilon}{I_{hf}}\right) \cdot \frac{N}{Z_0 + NR} \Delta R_{osc} I_{dc}$$

typically :

linewidth w^2 = 10 MHz, dispersion D= 100 MHz, I_{dc} = 5 mA

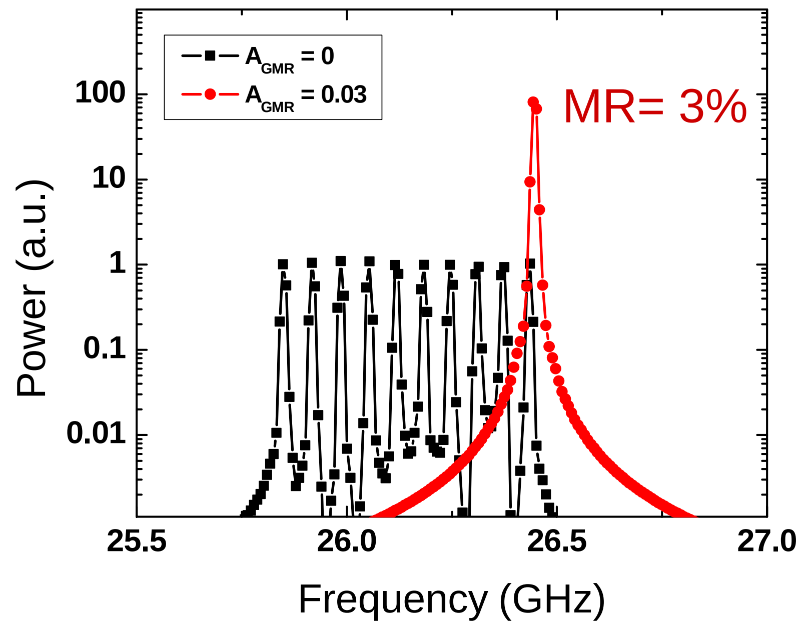
agility df/dI = 1 GHz/mA

condition $\frac{\Delta R_{osc}}{R} = 15\ \%$

agility df/dI = 10 GHz/mA (macrospin)

condition $\frac{\Delta R_{osc}}{R} = 1.6\ \%$

macrospin simulations



J. Grollier *et al*. PRB 73 060409 (R) 2006

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THALES

N STNOs connected in series

$$P = \frac{Z_0 N^2}{(Z_0 + NR)^2} \Delta R^2 I^2_{dc}$$

Case 1 : $Z_0 = 50\Omega$

if $NR \ll Z_0$

$$P \approx \frac{N^2}{Z_0} \Delta R^2 I^2_{dc}$$

Increases as N^2

if $NR \gg Z_0$

$$P \approx \frac{Z_0}{R^2} \Delta R^2 I^2_{dc}$$

Independant of N

Case 2 : $Z_0 = 10NR$

$$P \approx \frac{N}{10R} \Delta R^2 I^2_{dc}$$

Increases as N

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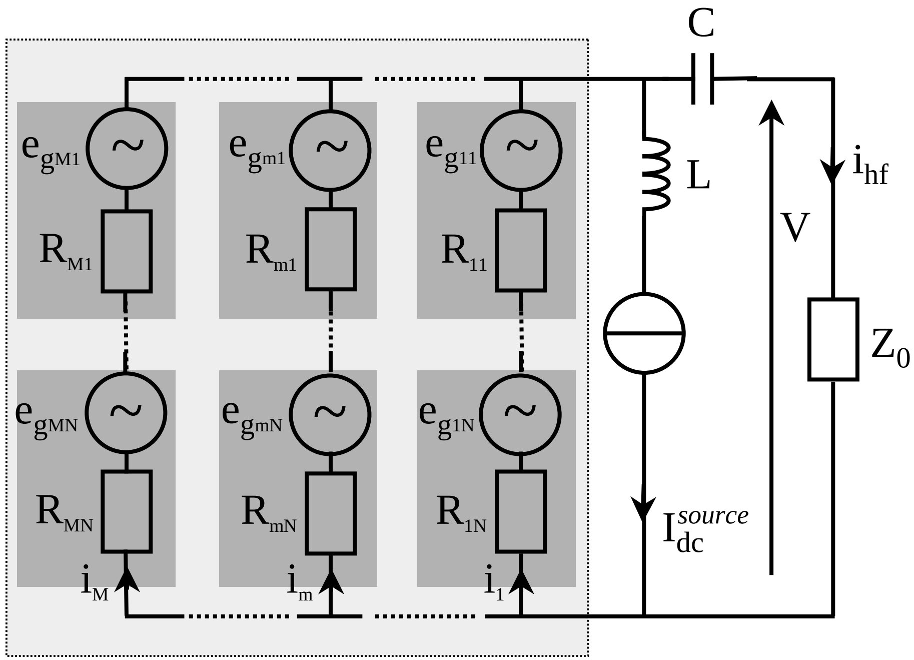
THALES

Best solution : hybrid systems = parallel + series connection

For impedance adaptation



In all cases P increases as N

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THALES

➢ Spin transfer oscillators : interesting features/physics

Spintronics / Non-linear dynamics

➢ low power : need to **synchronize** arrays

➢ conditions for synchronization using the results of phase locking experiments

B. Georges *et al.*, PRL **101,** 017201 (2008)

B. Georges *et al.*, APL **92**, 232504 (2008)

Phase locking to an external source : principle

General equation of the phase dynamics of a forced oscillator:

R. Adler, *IEEE*, 61, **10** (1973)

$$\frac{d(\Delta\Phi)}{dt} = (\omega_{free} - \omega_{source}) + \varepsilon \sin(\Delta\Phi) + \xi(t)$$

detuning: $(\omega_{free} - \omega_{source})$

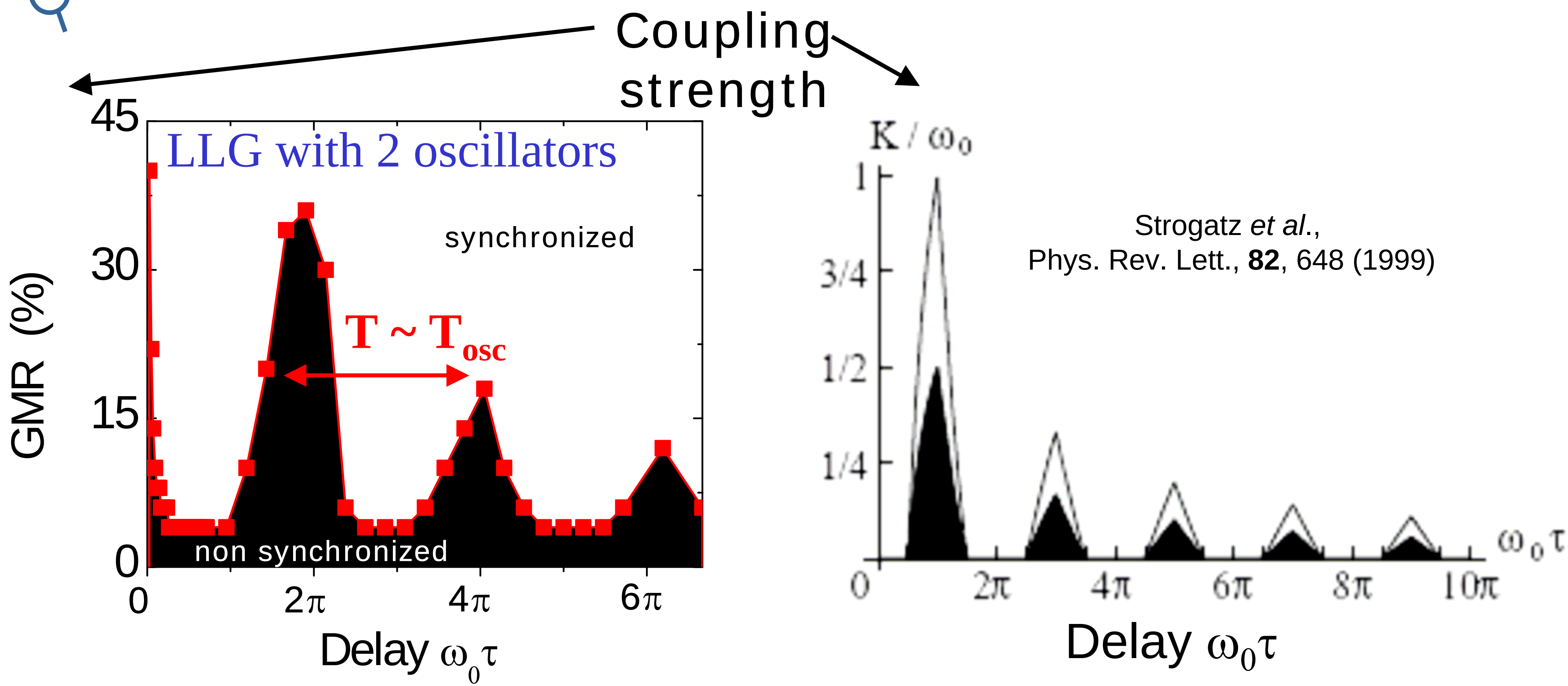
coupling strength: ε

noise σ^2: $\xi(t)$

- if $\varepsilon = 0$, $\Delta\Phi = (\omega_{free} - \omega_{source})t + \Phi_0$
- if $\varepsilon \neq 0$, $\Delta\Phi$ is alternatively increased and decreased due to the fact that $I_{tot} = I_{dc} + I_{hf} \sin(\omega_{source} t)$
- if ε > detuning there is a solution with constant $\Delta\Phi$ → locking
- the noise accounts for frequency fluctuations

if $\omega_0 < \omega$

Influence of delay on synchronization: simulation



- Coupled STOs can be understood in the frame of classical synchronization theory
- Delay is a key parameter in the design of arrays of STOs

THALES

Condition of phase locking

Phase dynamics of a forced STNO (from A. Slavin model)

$$\frac{d\left(\phi - \omega_{source} t\right)}{dt} = \omega_{FREE} - \omega_{source} + \varepsilon \cos\left(\omega_{source} t - \phi + \phi_0 + \frac{\pi}{2\sigma}\frac{\partial f_{FREE}}{\partial I_{dc}}\left(\frac{I_{dc}}{I_{th}} - 1\right)\right)$$

with

$$\varepsilon = \frac{I_{hf}}{2\sqrt{2}}\sigma \tan(\gamma)\sqrt{\frac{I_{dc}}{I_{dc} - I_{th}}}\sqrt{1 + \left(\frac{2\pi}{\sigma}\frac{I_{dc}}{I_{th}}\frac{\partial f_0}{\partial I_{dc}}\right)^2}$$

While locked to the source:

$$\phi = \omega_{source} t + \phi_0 + \frac{\pi}{2\sigma}\frac{\partial f_{FREE}}{\partial I_{dc}}\left(\frac{I_{dc}}{I_{th}} - 1\right) - ar\cos\left(\frac{2\pi\left(f_{source} - f_{FREE}\right)}{\varepsilon}\right)$$

THALES

While locked to the source :

$$\phi = \omega_{source} t + \phi_0 + \frac{\pi}{2\sigma} \frac{\partial f_{FREE}}{\partial I_{dc}} \left(\frac{I_{dc}}{I_{th}} - 1 \right) - ar\cos\left(\frac{2\pi\left(f_{source} - f_{FREE}\right)}{\varepsilon} \right)$$

If detuning = 0 :

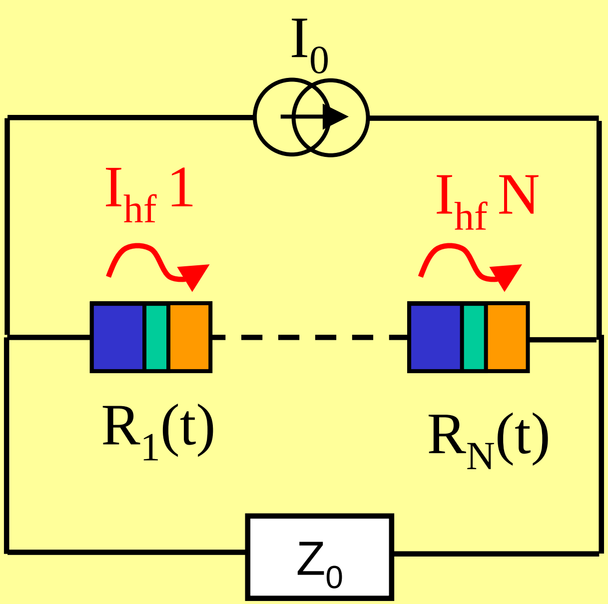
$$\phi - \omega_{source} t = \phi_0 + \frac{\pi}{2\sigma} \frac{\partial f_{FREE}}{\partial I_{dc}} \left(\frac{I_{dc}}{I_{th}} - 1 \right) - \frac{\pi}{2}$$

From the Adler equation to the Kuramoto model

1 STNO : Adler

$$\frac{d\Phi}{dt} = -2\pi\Delta f + \left(\frac{\varepsilon}{I_{hf}}\right).I_{hf}.\cos\Phi + \xi(t)$$

N STNOs connected in series



Kuramoto model

$$\frac{d\Phi_n}{dt} = -2\pi\Delta f_n + \frac{K}{N}\sum_n \cos\Phi_n + \xi_n(t)$$

Typically :

linewidth D = 10 MHz, I_{dc} = 5 mA, agility df/dI = 1GHz/mA

+ frequency dispersion αγιλιτψ,

condition $\frac{\Delta R_{osc}}{R} = 15\ \%$

+ optimistic value αγιλιτψ

condition $\frac{\Delta R_{osc}}{R} = 2.6\ \%$

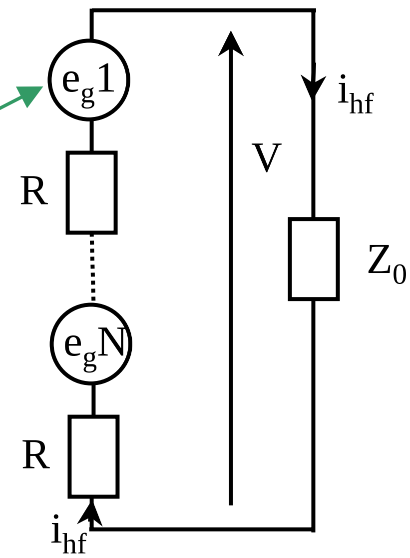
1 STNO : Adler

$$\frac{d\Phi}{dt} = -2\pi\Delta f + \left(\frac{\varepsilon}{I_{hf}}\right).I_{hf}.\cos\Phi + \xi(t)$$

N STNOs connected in series : **Kuramoto model**

$$i_{hf} = \frac{\Delta R_{osc}\, I_{dc}}{Z_0 + NR}\sum_n \cos\Phi_n$$

$e_g i = \Delta R_{osc}\, I_{dc} \cos \Phi_i$



$$\frac{d\Phi_n}{dt} = -2\pi\Delta f_n + \frac{K}{N}\sum_n \cos\Phi_n + \xi_n(t)$$

Kuramoto model with :

$$K = \left(\frac{\varepsilon}{I_{hf}}\right).\frac{N}{Z_0 + NR}\Delta R_{osc}\, I_{dc}$$

- analytical resolution
- assumptions : - lorentzian frequency dispersion (γ)
  - white noise (linewidth D)

Synchronization threshold $K_c = 2(\gamma + D)$

THALES

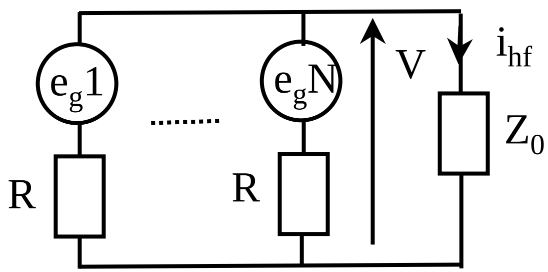
N STOs connected in series

$$P = Z_0 i^2_{hf} = \frac{Z_0 N^2}{(Z_0 + NR)^2} \Delta R^2 \, I^2_{dc}$$

if all are synchronized

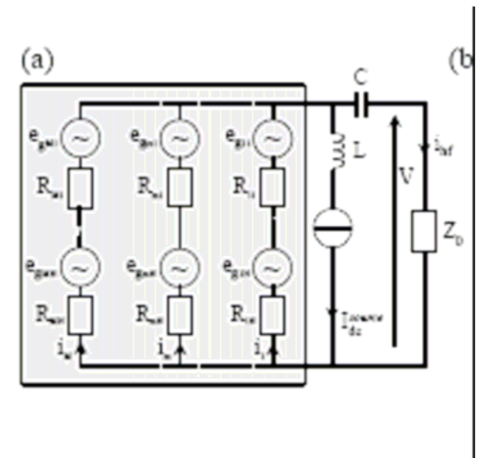
Usually Z_0=50 Ω

N STOs connected in // :



$$P = Z_0 i^2_{hf} = \frac{Z_0 N^2}{(Z_0 N + R)^2} \Delta R^2 \, I^2_{dc}$$

if all are synchronized

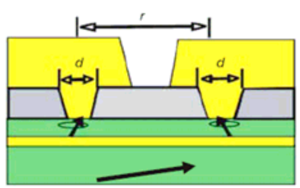


Best solution : hybrid systems // + series connection

For impedance adaptation

In all cases P increases as N

NIST



Synchronization by SW : can work,
But pb with emitted power !

THALES

Mutual synchronization: simulation

➢Analytical expression for the phase dynamics of coupled STOs from LLG

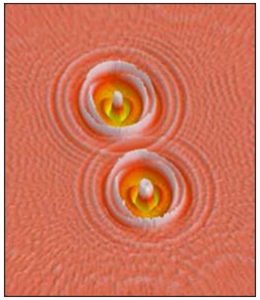
$$\langle \dot{\varphi}_i \rangle = \omega_i - \frac{1}{2}\frac{\gamma_0}{1+\alpha^2} J_{dc}\beta(GMR)\left[1-\left(\frac{H}{H_d}\right)^2\right]\sum_{j=1}^{N}\sin(\varphi_j-\varphi_i)$$

Kuramoto model

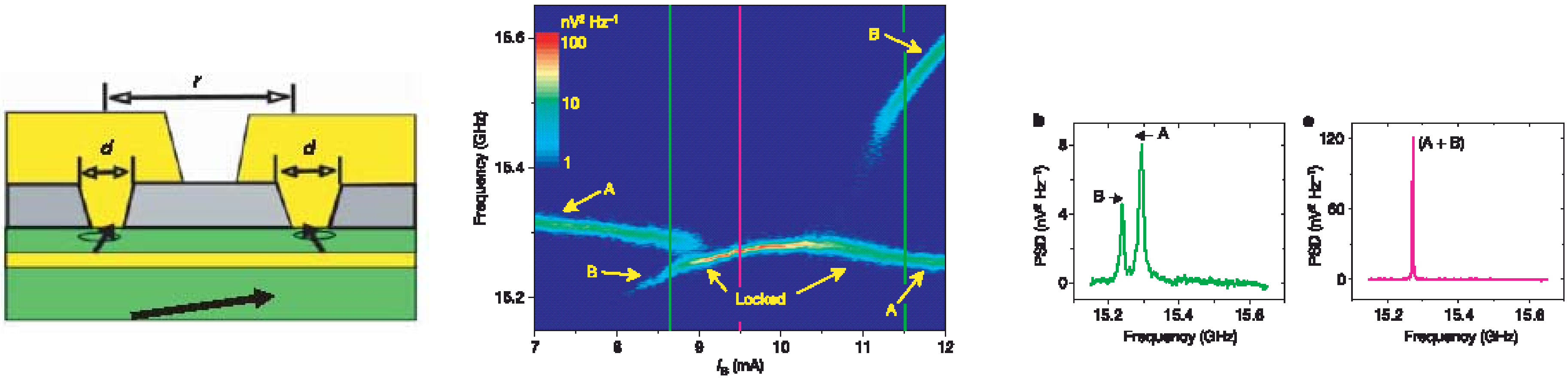
$$\dot{\varphi}_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\varphi_j-\varphi_i)$$

Influence of the delay : length of the wire

Different kinds of couplings in STOs (1)

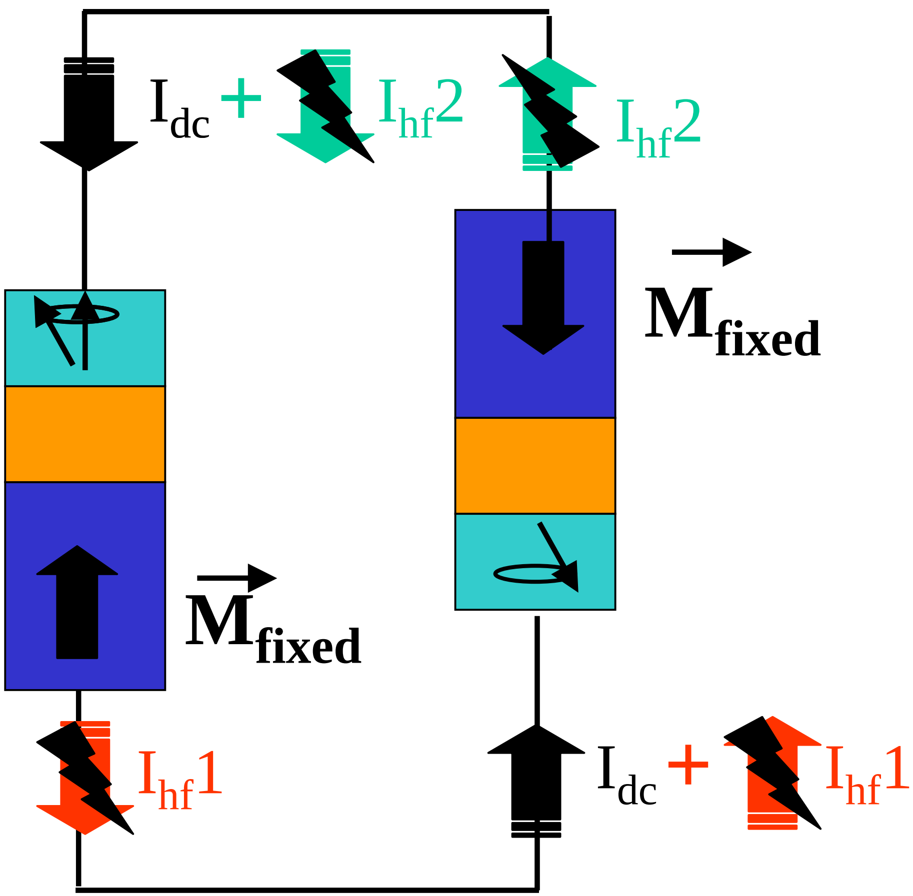


Spin waves coupling (local)



Figures from Kaka *et al.*, Nature 2005

Coupling via self-emitted microwave current



$I_{hf} \propto \Delta R$

coupling strength $\propto I_{hf}$

Global coupling

Current through one oscillator

$$I_{tot} = I_{dc} + \sum_{i=1}^{N} I_{hf}(i)$$

$f = f(I_{tot})$ → f_o, Δf

J. Grollier et al. PRB 73 060409 (R) 2006