Phase locking of a Spin Transfer Nano-Oscillator to an external microwave current:

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The LLG equation is given by:

\[
\frac{d\vec{m}}{dt} = -\gamma_0 \vec{m} \times \vec{H}_{\text{eff}} + \alpha \vec{m} \times \frac{d\vec{m}}{dt} + \gamma_0 J \vec{m} \times (\vec{m} \times \vec{M})
\]

Precession if:

\[R_0 - R_s = 0\]

Spin torque compensates the damping

For \( I > 0 \):

\[\left(\frac{d\vec{m}}{dt}\right)_{\text{inj}}\]

Dissipative resistance:

\[R_0 \propto \alpha_G \omega\]

Negative resistance:

\[-R_s \propto \gamma_0 J\]

A. Slavin model *IEEE 41*, 4 (2005)
\[ V(t) = \Delta R_{\text{max}} I_{dc} \bar{m}(t) \cdot \bar{M} \]

\[ P \approx \frac{\Delta R^2_{\text{osc}} I_{dc}^2}{Z_{\text{LOAD}}} \]

Intensity (dBm) vs. Frequency (GHz) for T = 25 K, \( \Delta f = 10\text{MHz} \), f = 2.21 GHz.
Spin Transfer Nano-Oscillators

- non-linear \((I_{dc} \rightarrow V_{ac})\)
- agile \((I_{do}, H)\)
- direct emission up to 40 GHz
- high Q factor
- sub-micron size

Applications:
- Telecommunications
- RADAR

Problem: very weak emitted power

\[ P_{out} \sim -60 \text{ dBm} \]

Too low for applications

Solution: synchronization

Coherent emission in frequency and phase of many oscillators
Synchronization: the oldest non-linear effect ever studied!!

- Other examples: crickets, applauding audience, pace maker cells, intestine cellulls, circadian rythm, Josephson junctions etc.
Couplings in STO devices

Non local coupling:
self-emitted microwave currents

Local coupling:
spin waves or dipolar fields

\[ I_0 \]

\[ I_{hf \, 1} \]

\[ I_{hf \, N} \]

\[ R_1(t) \]

\[ R_N(t) \]
First step: phase locking to a microwave source

Electrically connected oscillators

Current through ONE oscillator

\[ I_{tot} = I_{dc} + \sum_{i=1}^{N} I_{hf}(i) \]

- \( f_0 \)
- \( \Delta f, \Delta \phi \)

Injection of a microwave signal

Current through THE oscillator

\[ I_{tot} = I_{dc} + I_{hf} \text{ source} \]

- \( f_0 \)
- \( \Delta f, \Delta \phi \)
Features of a single STNO

- Microwave characteristics of the emission of an STNO:
  - Frequency
  - Linewidth
  - Power
  - Agility in current \( (\frac{df_{FREE}}{dI_{dc}}) \)

Change with the experimental conditions (H and I_{dc})
Features of a single STNO

microwave characteristics of the emission of an STNO:

- frequency
- linewidth
- power
- agility in current (df_{FREE}/dI_{dc})

change with the experimental conditions (H and I_{dc})
Phase locking to an external source

$I_{dc}$

$f_{FREE}$ (GHz)

$H = -2.60 \text{ kOe}$

$\text{pW/GHz/mA}^2$

$\text{without source}$
Phase locking to an external source

$\mathbf{I}_{\text{dc}}$ $\sim \mathbf{I}_{\text{hf}}$

\begin{align*}
\text{without source} & : H = -2.60 \text{ kOe} \\
\text{with } I_{\text{hf}} = 1 \text{ mA} & : \text{shifted and locked}
\end{align*}

\begin{align*}
\mathbf{f}_{\text{FREE}} & (\text{GHz}) \\
\mathbf{f}_{\text{FORCED}} & (\text{GHz})
\end{align*}

\begin{align*}
\mathbf{f}_{\text{source}} & (\text{GHz})
\end{align*}
Locking range and frequency shift

\((H, I_{dc})\) chosen so that only the agility in current changes.

The microwave characteristics of the STNO determine the **coupling strength** to the external microwave current.
The microwave characteristics of the STNO determine the coupling strength to the external microwave current.
Synchronization / Phase dynamics (100 oscillators)

Macrospin simulations

 SYNC : adjustment of the PHASE

J = 0.035 T
τ = 5 ps
A_{GMR} = 0.03

H_{an} (T)

J = 0.01 T
τ = 300 ps
A_{GMR} = 0.4
Derivation of the phase dynamics

**equation of the magnetization motion**

\[
\frac{db}{dt} = -i(\omega_{FMR} + Nb^2)b - \Gamma(1 - Qb^2)b + \sigma I_{dc}(1 - b^2)b
\]

- rotation
- damping
- spin transfer

\[b = ce^{i\phi}\]

Spin-wave: amplitude \(c\) and phase \(\phi\)


**uniformly rotating phase**

\[
\phi = \varphi + \frac{N}{\sigma I_{dc} + \Gamma Q}\ln(c) + \phi_0
\]

valid even when slightly perturbed

Derivation of the phase dynamics

\[
\frac{db}{dt} = -i(\omega_{FMR} + Nb^2)b - \Gamma(1 - Qb^2)b + \sigma I_{dc}(1 - b^2)b + \frac{\sigma I_{hf}}{2\sqrt{2}} \tan(\gamma)e^{-i\sigma_s t}
\]

external source term

phase dynamics:

\[
\frac{d(\Delta \phi)}{dt} = 2\pi(f_{FREE} - f_{source}) + \varepsilon \sin(\Delta \phi) + \xi(t)
\]

detuning
coupling strength
noise \(w^2\)

Adler equation

R. Adler, *IEEE, 61, 10* (1973)

coupling strength

\[
\varepsilon = \frac{I_{hf}}{2\sqrt{2}} \sigma \tan(\gamma) \sqrt{\frac{I_{dc}}{I_{dc} - I_{th}}} \sqrt{1 + \left(\frac{2\pi I_{dc}}{\sigma I_{th}} \frac{\partial f_{FREE}}{\partial I_{dc}}\right)^2}
\]

Phase locking to an external source: principle

General equation of the phase dynamics of a forced oscillator:

\[ \frac{d(\Delta \Phi)}{dt} = (\omega_{\text{free}} - \omega_{\text{source}}). \]

Detuning

- If \( \varepsilon = 0 \)
  \[ \Delta \Phi = (\omega_{\text{free}} - \omega_{\text{source}}) t + \Phi_0 \]

the STNO and the source evolve independently
Phase locking to an external source: principle

General equation of the phase dynamics of a forced oscillator:

\[
\frac{d(\Delta \Phi)}{dt} = (\omega_{\text{free}} - \omega_{\text{source}}) + \varepsilon \sin(\Delta \Phi)
\]

- **if** \( \varepsilon \neq 0 \)
- \( \Delta \Phi \) is alternatively increased and decreased

Indeed: \( I_{\text{tot}} = I_{\text{dc}} + I_{\text{hf}} \sin(\omega_{\text{source}} t) \)

\( \omega_{\text{free}} \) is alternatively increased and decreased

\( \varepsilon \) small

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R. Adler, *IEEE*, 61, 10 (1973)
Phase locking to an external source: principle

General equation of the phase dynamics of a forced oscillator:

\[
\frac{d(\Delta \Phi)}{dt} = (\omega_{\text{free}} - \omega_{\text{source}}) + \varepsilon \sin(\Delta \Phi)
\]

- if \( \varepsilon > \) detuning
  - there is a solution with constant \( \Delta \Phi \)

locking for large couplings
Phase locking to an external source: principle

General equation of the phase dynamics of a forced oscillator:

\[
\frac{d(\Delta \Phi)}{dt} = (\omega_{\text{free}} - \omega_{\text{source}}) + \varepsilon \sin(\Delta \Phi) + \xi(t)
\]

- the noise $\xi$ accounts for frequency fluctuations (linewidth)

R. Adler, *IEEE*, 61, 10 (1973)
mismatch = $f_{\text{FORCED}} - f_{\text{source}}$

detuning = $f_{\text{FREE}} - f_{\text{source}}$

$I_{\text{hf}} = 0\, \text{mA}$
The influence of phase noise: experiment vs model

\[ \frac{d(\Delta \phi)}{dt} = 2\pi (f_{\text{FREE}} - f_{\text{source}}) + \varepsilon \sin(\Delta \phi) + \xi(t) \]

The calculation of the mismatch depends on:
- detuning \( (f_{\text{FREE}} - f_{\text{source}}) \)
- noise \( w^2 \) (linewidth)
- coupling strength \( \varepsilon \) (free parameter)

The experimental data shows a linear relationship between mismatch and detuning with a slope of 1 MHz/mA for \( I_{hf} = 1 \text{mA} \).
Influence of phase noise: experiment vs model

\[ \frac{d(\Delta \phi)}{dt} = 2\pi(f_{\text{FREE}} - f_{\text{source}}) + \varepsilon \sin(\Delta \phi) + \xi(t) \]

calculation of the mismatch depends on:

- detuning \((f_{\text{FREE}} - f_{\text{source}})\)
- noise \( w^2 \) (linewidth)
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Influence of phase noise: experiment vs model

\[ \frac{d(\Delta \phi)}{dt} = 2\pi (f_{\text{FREE}} - f_{\text{source}}) + \varepsilon \sin(\Delta \phi) + \xi(t) \]

Calculation of the mismatch depends on:
- detuning \( (f_{\text{FREE}} - f_{\text{source}}) \)
- noise \( w^2 \) (linewidth)
- coupling strength \( \varepsilon \) (free parameter)

Experimental data \( (w^2 = 17 \text{ MHz}) \)
Calculation \( (w^2 = 0 \text{ MHz}) \)
Calculation \( (w^2 = 17 \text{ MHz}) \)

\( \varepsilon = 25 \text{ MHz} \)
\( I_{hf} = 1 \text{ mA} \)
Forced STNOs can be described with the Adler model of phase locking.

Critical influence of phase noise:

- detuning ($f_{\text{FREE}} - f_{\text{source}}$)
- noise $w^2$ (linewidth)
- coupling strength $\varepsilon$ (free parameter)

Expected values:

- Experimental data ($w^2 = 17$ MHz)
- Calculation ($w^2 = 0$ MHz)
- Calculation ($w^2 = 17$ MHz)

Influence of phase noise: experiment vs model depends on:

$\frac{d(\Delta \phi)}{dt} = 2\pi (f_{\text{FREE}} - f_{\text{source}}) + \varepsilon \sin(\Delta \phi) + \xi(t)$
Experimental test of the coupling calculation

\[ \varepsilon = \frac{I_{hf}}{2\sqrt{2}} \sqrt{\frac{I_{dc}}{I_{dc} - I_{th}}} \sqrt{1 + \left( \frac{2\pi I_{dc} \frac{\partial f_{\text{FREE}}}{\partial I_{dc}}}{\sigma I_{th}} \right)^2} \]

with \( \sigma = 825 \text{ MHz/mA} \)

\( \Rightarrow \gamma = 2.75^\circ \)
Experimental test of the coupling calculation

\[ \varepsilon = I_{hf} \left( \frac{\pi \tan(\gamma)}{\sqrt{2}} \right) \sqrt{\frac{I_{dc}}{I_{dc} - I_{th}}} \sqrt{1 + \left( \frac{2\pi}{\sigma I_{th}} \frac{\partial f_{FREE}}{\partial I_{dc}} \right)^2} \]

- **Constant**
- **Varying**

**Figure:**
- Left: Graph showing experimental data and linear fit with 30MHz/mA.
- Right: Scatter plot for \( \varepsilon / I_{hf} \) showing experimental data with error bars.
Experimental test of the coupling calculation

\[ \varepsilon = I_{hf} \frac{\sigma \tan(\gamma)}{2\sqrt{2}} \sqrt{\frac{I_{dc}}{I_{dc} - I_{th}}} \left( 1 + \frac{2\pi I_{dc}}{\sigma I_{th}} \frac{\partial f_{FREE}}{\partial I_{dc}} \right)^2 \]

with \( \sigma = 825 \text{ MHz/mA (s=0.31)} \)

\[ \Rightarrow \gamma = 2.75^\circ \]

High agility enhances the coupling
qualitative and quantitative understanding of injection locking experiments:

- description with Adler model of forced oscillators
- expression of the coupling strength

**main trends:**

✓ bad influence of the noise on phase locking
✓ high agility enhances the coupling

From Adler equation to the Kuramoto model

**Adler equation**

\[
\frac{d(\Delta \phi)}{dt} = 2\pi(f_{FREE} - f_{source}) + \varepsilon \sin(\Delta \phi) + \xi(t)
\]

**Kuramoto model**

\[
\frac{d\Delta \phi_n}{dt} = 2\pi f_n + \frac{K}{N} \sum_n \cos \Delta \phi_n + \xi_n(t)
\]

**From Adler equation to the Kuramoto model**

**forced STNO**

- \( I_{dc} \)
- \( I_{hf1} \)
- \( I_{hf source} \)
- \( R_1(t) \)
- \( Z_0 \)

**N STNOs connected in series**

- \( I_{dc} \)
- \( I_{hf1} \)
- \( I_{hf2} \)
- \( I_{hfN} \)
- \( R_1(t) \)
- \( R_N(t) \)
- \( Z_0 \)

\[
\varepsilon = \frac{\sigma \tan(\gamma)}{2\sqrt{2}} \left[ \frac{I_{dc}}{I_{dc} - I_{th}} \right]^{\frac{1}{2}} + \left( \frac{2\pi I_{dc} \frac{\partial f_{FREE}}{\partial I_{dc}}}{\sigma I_{th} \frac{\partial I_{dc}}{\partial I_{dc}}} \right)^2
\]

\[
K = \left( \frac{\varepsilon}{I_{hf}} \right) \cdot \frac{N}{Z_0 + NR} \Delta R_{osc} I_{dc}
\]
Synchronization threshold $K_c = 2(w^2 + D)$

typically:
linewidth $w^2 = 10$ MHz, dispersion $D = 100$ MHz, $I_{dc} = 5$ mA

agility $df/dI = 1$ GHz/mA

condition

\[ \frac{\Delta R_{osc}}{R} = 15\% \]

agility $df/dI = 10$ GHz/mA (macrospin)

condition

\[ \frac{\Delta R_{osc}}{R} = 1.6\% \]

macrosspin simulations

J. Grollier et al. PRB 73 060409 (R) 2006
Discussion emitted power (1)

N STNOs connected in series

\[ P = \frac{Z_0 N^2 \Delta R^2 I^2_{dc}}{(Z_0 + NR)^2} \]

Case 1: \( Z_0 = 50\Omega \)

- if \( NR \ll Z_0 \)
  \[ P \approx \frac{N^2}{Z_0} \Delta R^2 I^2_{dc} \]
  Increases as \( N^2 \)

- if \( NR \gg Z_0 \)
  \[ P \approx \frac{Z_0}{R^2} \Delta R^2 I^2_{dc} \]
  Independant of \( N \)

Case 2: \( Z_0 = 10NR \)

\[ P \approx \frac{N}{10R} \Delta R^2 I^2_{dc} \]

Increases as \( N \)
Best solution: hybrid systems = parallel + series connection

For impedance adaptation

In all cases $P$ increases as $N$
Spin transfer oscillators: interesting features/physics

Spintronics / Non-linear dynamics

- low power: need to **synchronize** arrays

- **conditions for synchronization** using the results of phase locking experiments

Phase locking to an external source: principle

General equation of the phase dynamics of a forced oscillator:

\[ \frac{d(\Delta \Phi)}{dt} = (\omega_{\text{free}} - \omega_{\text{source}}) + \varepsilon \sin(\Delta \Phi) + \xi(t) \]

- if \( \varepsilon = 0 \), \( \Delta \Phi = (\omega_{\text{free}} - \omega_{\text{source}}) t + \Phi_0 \)
- if \( \varepsilon \neq 0 \), \( \Delta \Phi \) is alternatively increased and decreased due to the fact that \( I_{\text{tot}} = I_{dc} + I_{hf} \sin(\omega_{\text{source}} t) \)
- if \( \varepsilon > \) detuning there is a solution with constant \( \Delta \Phi \) locking
- the noise accounts for frequency fluctuations

if \( \omega_0 < \omega \)
Coupled STOs can be understood in the frame of classical synchronization theory. Delay is a key parameter in the design of arrays of STOs.

Influence of delay on synchronization: simulation

Phase dynamics of a forced STNO (from A. Slavin model)

\[
\frac{d(\phi - \omega_{\text{source}} t)}{dt} = \omega_{\text{FREE}} - \omega_{\text{source}} + \varepsilon \cos \left( \omega_{\text{source}} t - \phi + \phi_0 + \frac{\pi}{2\sigma} \frac{\partial f_{\text{FREE}}}{\partial I_{dc}} \left( \frac{I_{dc}}{I_{th}} - 1 \right) \right)
\]

with

\[
\varepsilon = \frac{I_{hf}}{2\sqrt{2}} \sigma \tan(\gamma) \sqrt{\frac{I_{dc}}{I_{dc} - I_{th}}} \sqrt{1 + \left( \frac{2\pi}{\sigma} \frac{I_{dc}}{I_{th}} \frac{\partial f_0}{\partial I_{dc}} \right)^2}
\]

While locked to the source:

\[
\phi = \omega_{\text{source}} t + \phi_0 + \frac{\pi}{2\sigma} \frac{\partial f_{\text{FREE}}}{\partial I_{dc}} \left( \frac{I_{dc}}{I_{th}} - 1 \right) - \arccos \left( \frac{2\pi (f_{\text{source}} - f_{\text{FREE}})}{\varepsilon} \right)
\]
Condition of phase locking

While locked to the source:

$$\phi = \omega_{source} \ t + \phi_0 + \frac{\pi}{2\sigma} \frac{\partial f_{FREE}}{\partial I_{dc}} \left( \frac{I_{dc}}{I_{th}} - 1 \right) - \arccos \left( \frac{2\pi \left( f_{source} - f_{FREE} \right)}{\varepsilon} \right)$$

If detuning = 0:

$$\phi - \omega_{source} \ t = \phi_0 + \frac{\pi}{2\sigma} \frac{\partial f_{FREE}}{\partial I_{dc}} \left( \frac{I_{dc}}{I_{th}} - 1 \right) - \frac{\pi}{2}$$
From the Adler equation to the Kuramoto model

1 STNO : Adler

$$\frac{d\Phi}{dt} = -2\pi \Delta f + \left( \frac{\varepsilon}{I_{hf}} \right) I_{hf} \cos \Phi + \xi(t)$$

N STNOs connected in series

Kuramoto model

$$\frac{d\Phi_n}{dt} = -2\pi \Delta f_n + \frac{K}{N} \sum_n \cos \Phi_n + \xi_n(t)$$

Typically:
linewidth $D = 10$ MHz, $I_{dc} = 5$ mA, agility $df/dI = 1$GHz/mA

+ frequency dispersion $\alpha \gamma \iota \lambda \iota \tau \psi$,

condition

$$\frac{\Delta R_{osc}}{R} = 15\%$$

+ optimistic value $\alpha \gamma \iota \lambda \iota \tau \psi$

condition

$$\frac{\Delta R_{osc}}{R} = 2.6\%$$
From the Adler equation to the Kuramoto model

1 STNO : Adler

\[ \frac{d\Phi}{dt} = -2\pi\Delta f + \left( \frac{\varepsilon}{I_{hf}} \right) I_{hf} \cdot \cos \Phi + \xi(t) \]

N STNOs connected in series : Kuramoto model

\[ i_{hf} = \frac{\Delta R_{osc} I_{dc}}{Z_0 + NR} \sum_n \cos \Phi_n \]

\[ \frac{d\Phi_n}{dt} = -2\pi\Delta f_n + \frac{K}{N} \sum_n \cos \Phi_n + \xi_n(t) \]

Kuramoto model with :

- analytical resolution
- assumptions : - lorentzian frequency dispersion (\(\gamma\))
  - white noise (linewidth D)

**Synchronization threshold** \(K_c = 2(\gamma+D)\)
Emitted power

\[
P = Z_0 i_{hf}^2 = \frac{Z_0 N^2}{(Z_0 + N R)^2} \Delta R^2 I_{dc}^2
\]

Usually \( Z_0 = 50 \, \Omega \)

N STOs connected in series

if all are synchronized

N STOs connected in \( /// \) :

Best solution: hybrid systems \( /// \) + series connection

For impedance adaptation

In all cases \( P \) increases as \( N \)

Synchronization by SW: can work, But pb with emitted power!
Mutual synchronization: simulation

Analytical expression for the phase dynamics of coupled STOs from LLG:

\[ \dot{\varphi}_i = \omega_i - \frac{1}{2} \frac{\gamma_0}{\alpha^2} J_{dc} \beta(GMR) \left[ 1 - \left( \frac{H}{H_d} \right)^2 \right] \sum_{j=1}^{N} \sin(\varphi_j - \varphi_i) \]

Kuramoto model:

\[ \dot{\varphi}_i = \omega_i + \frac{K}{N} \sum_{j=1}^{N} \sin(\varphi_j - \varphi_i) \]

Influence of the delay: length of the wire

\[ I_{dc}, \quad I_{hf1}(t), \quad I_{hf1}(t+\tau) \]
Spin waves coupling (local)

Figures from Kaka et al., Nature 2005
Different kinds of couplings in STOs (2)

Coupling via self-emitted microwave current

Global coupling

Current through one oscillator

\[ I_{tot} = I_{dc} + \sum_{i=1}^{N} I_{hf}^{(i)} \]

\[ f = f(I_{tot}) \]

J. Grollier et al. PRB 73 060409 (R) 2006