Current-induced spin-transfer torque in magnetic tunnel junctions

M. Wilczyński¹, R. Swirkowicz¹, J. Barnaś²

 ¹Faculty of Physics, Warsaw University of Technology, Koszykowa 75, 00-662 Warsaw, Poland
 ²Institute of Molecular Physics, Polish Academy of Sciences, Smoluchowskiego 17, 60-179 Poznań, Poland

Outline

- Origin of the spin transfer torque
- Spin torque in a single tunnel junction with semiinfinite electrodes
- Angular-dependence
- Bias-dependence
- Spin torque in non-symmetric MTJ
- STT in junctions with a ferromagnetic layer of a finite thickness
- Spin torque in double planar junctions
- Conclusions

Spin transfer torque

Spin transfer torque arises when spin polarized current is filtered by magnetic layer whose moment is non collinear with spin of conduction electrons. In the process of filtering the magnet absorbs a portion of spin angular momentum that is carried out by electron spins. The magnetization of the ferromagnet exerts a torque on the flowing spins to change their orientation and the flowing electrons exert an equal but opposite torque on the ferromagnet. The torque exerted by conduction electrons is called spin transfer torque (predicted by Slonczewski and Berger).



Current induced switching



NiFe/Cu/NiFe (Braganca et al. 2005



Two qualitatively different types of magnetic behaviour can be observed:
a dynamical state in which the magnetization undergoes

- steady-state precession
- switching from one static magnetic orientation to another

CoFeB/MgO/CoFeB (Sun et al.)

When the current density is sufficiently high the switching of the magnetization can be obtained. The effect has been confirmed by many experiments both in metallic multilayers and in tunnel junctions **It offers a possibility of manipulating magnetic device elements without applying magnetic field.**



Voltage oscillations produced by steady precession of the magnetic free layer in nanopillar sample NiFe/Cu/NiFe (Krivorotov(2005)

The current can induce a precession of the magnetic moment in the microwave range

Schematic geometry

The simplest system in which the torque can be observed consists of two magnetic layers separated by a thin non-magnetic spacer.







Spin torque driven switching

The current sufficient to switch the magnetic moment of the free layer (the critical current) can be lower than 10^7 A/cm² for devices with metallic non-magnetic spacer. In tunnel junction with a non-magnetic MgO based barrier it is of the order of 10^6 A/cm².

$$J_{\rm c0} = \frac{2e\alpha M_{\rm s} t_{\rm F} (H_K \pm H_{\rm ext} + 2\pi M_{\rm s})}{\hbar \eta}$$

Critical current depends on : the saturation magnetization M_s , damping constant α , thickness of the free layer t_F , effective anisotropy field H_K , external magnetic field $H_{ext,}$ spin transfer efficiency η

Magnetic tunnel junction CoFeB/MgO/CoFeB (Kubota et al. 2007)



all metal nanopillar sample NiFe/Cu/NiFe (Braganca et al. 2005)



$TMR = \frac{R_{AP} - R_P}{R_P}$ Spin torque driven switching in MTJ



The field hysteresis loop (a) and current loop (b) in $AIO_x MTJ$ cell. The nominal cell dimension is 127 x 148 nm² and the average critical current density $\langle J_{c0} \rangle = 6 \ 10^6 \ Acm^{-2}$ (Huai et al. 2006)

The field hysteresis loop (a) and current loop (b) in **MgO MTJ** cell. The nominal cell dimension is $125 \times 220 \text{ nm}^2$ and the average critical current density $<\mathbf{J}_{c0}>=2.2 \ 10^6 \ \mathrm{Acm}^{-2}$ (Huai et al. 2006)

The application of STT in MRAM type devices requires low switching current and high TMR ratio. Tunnel junctions are better suited than metallic multilayers for many types of applications as they show large TMR and they can be better matched to silicon-based electronics

Spin torque

Since angular momentum is conserved the spin transfer torque can be calculated by determining the net flux of spin current

$$\vec{T} = -\frac{\hbar}{2} \oint dR^2 \vec{n} \vec{\vec{J}}^s = -\frac{\hbar}{2} \int_{pillbox} dr^3 \nabla \vec{\vec{J}}^s$$

Total spin current is not conserved. The torque on a unit area of the ferromagnet is equal to the net spin current transferred from conduction electrons to ferromagnet

torque acting on the left semi-infinite layer

$$T_{\parallel}^{l} = -\frac{\hbar}{2}J_{x}^{sl} \qquad T_{\perp}^{l} = -\frac{\hbar}{2}J_{y}^{sl}$$

$$J_{x'}^{sr} = J_x^{sr} \cos\theta + J_z^{sr} \sin\theta$$



direction of tunneling for V>0

torque acting on the right layer

$$T_{\parallel}^{r} = \frac{\hbar}{2} \Big[J_{x'}^{sr} - \widetilde{J}_{x'}^{sr} \Big]$$

$$T_{\perp}^{r} = \frac{\hbar}{2} \left[J_{y}^{sr} - \widetilde{J}_{y}^{sr} \right]$$

The incoming spin/current

The out coming spin current

Spin current

The spin current density carried by one electron described by the wave function Ψ is given by

$$\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$$
 Pauli matrices

$$\vec{j}^s = \frac{\hbar}{m} \operatorname{Im}(\Psi^* \vec{\sigma} \otimes \nabla \Psi) \Psi =$$

 $\begin{bmatrix} \psi_{\uparrow}(y) \\ \psi_{\downarrow}(y) \end{bmatrix}$ components taken with respect to local reference frame

z-component is conserved and it is equal to

$$j_z^s(y) = \frac{\hbar}{m} \operatorname{Im} \sum_{\sigma'} \hat{\sigma}' \psi_{\sigma'}^* \left[\frac{d\psi_{\sigma'}}{dy} \right]$$

Transverse components are not conserved and are equal to

$$j_{x}^{s}(y) = \operatorname{Re} j_{+}^{s}(y) \quad j_{y}^{s}(y) = \operatorname{Im} j_{+}^{s}(y) \qquad j_{+}^{s}(y) = \frac{i\hbar}{m} \left[\frac{d\psi_{\uparrow}}{dy} \psi_{\downarrow} - \psi_{\uparrow}^{*} \frac{d\psi_{\downarrow}}{dy} \right]$$

charge current density
$$j(y) = \frac{e\hbar}{m} \operatorname{Im} \sum_{\sigma'} \psi_{\sigma'}^{*} \left[\frac{d\psi_{\sigma'}}{dy} \right]$$

Free-electron-like model

Electronic structure of ferromagnetic layers is modelled by spin split parabolic bands Calculations are limited to zero temperature. We used the fact that electron transport in MTJ is mainly ballistic.



Wave function of tunnelling electron

The wave function of the electron propagating along y axis is expressed by plane waves in ferromagnetic layers :

 $\psi_{\sigma'i}(y) = A_{i\sigma'} \exp(ik_{i\sigma'}y) + B_{i\sigma'} \exp(-ik_{i\sigma'}y)$ in the *i*-th electrode (*i*=*l*,*r*).

for electron of spin σ impinging from the left on the left barrier

arrier $A_{l\sigma} = 1$ $A_{l-\sigma} = 0$ $B_{r\sigma} = 0$ $B_{r-\sigma} = 0$

 $\psi_{\sigma'B}(y) = C_{B\sigma'}Ai(Z) + D_{B\sigma'}Bi(Z)$ in the barrier

 $k_{i\sigma'}$ is the component of the wave vector normal to electrode/barrier interface, Ai, Bi -Airy functions Z = f(y)

Coefficients $A_{i\sigma'}, B_{i\sigma'}, C_{B\sigma'}, D_{B\sigma'}$ are found according to the matching conditions of the wave function and its first derivative FM Tunnel barrier (I) FM

The matching conditions at the barrier/right FM interface take the form:

$$\psi_{\downarrow B} = -\psi_{\uparrow r} \sin\left(\frac{\theta}{2}\right) + \psi_{\downarrow r} \cos\left(\frac{\theta}{2}\right)$$
$$\psi_{\uparrow B} = \psi_{\uparrow r} \cos\left(\frac{\theta}{2}\right) + \psi_{\downarrow r} \sin\left(\frac{\theta}{2}\right)$$



Total spin current density

The total spin current density is calculated by taking the sum over all occupied states

 E_F Fermi energy in the left electrode, $E_{l(r)\sigma}^b$ - position of the band bottom for the electrons of spin σ in the left (right) electrode, σ spin of the incident electron.

When the magnetization direction is non-uniform the spin currents carried by electrons moving from the left to the right and from the right to the left do not cancel and the net spin current can appear with no bias applied. Only normal component of spin current is different from zero with no bias applied

$$V = 0 \quad J^{s}_{\perp V=0} \neq 0 \quad j^{sl \to r}_{\perp} (\varepsilon_{\perp})_{V=0} = j^{sr \to l}_{\perp} (\varepsilon_{\perp})_{V=0} \quad J^{s}_{\parallel V=0} = 0$$

When the voltage is applied electrons with energy from the tunnel window contribute to the charge current and the spin current

Theoretical approaches to spin transfer torque in MTJ

Free-electron model as well as Bardeen Transfer Hamiltonian approach (Slonczewski)

Free-electron-like model based on WKB approximation and Green function formalism (e.g. Manchon et al.),

A tight binding model and the Green function method (Theodonis et al)

Scattering matrix approach (impurity scattering and interface roughness neglected) (Bauer, Brataas)

Ab initio approach based on Korringa-Kohn-Rostoker method with use of multiple scattering Green's function formalism, Fe/MgO/Fe tunnel junction (Heiliger, Stilles)

Simple model calculations show that in MTJ both components of the spin torque are of comparable magnitude. This is in contrast to metallic multillayer systems in which the normal component is practically equal to zero

Contributions summed over all relevant states on the Fermi surface in metallic systems cancel for the normal component and only the in-plane torque is different from zero.

In tunnel junctions the situation is different since tunneling is dominated by electrons coming from particular parts of the Fermi surface,

Components of the local torque oscillate with a different phase in a ferromagnetic layer with a distance from I/FM interface



Free-electron model. Numerical results

Parameters of the junction

Fermi energy $E_f = 2.62 eV$ half of the spin splitting of the electron band $\Delta = 1.96 eV$ height of the barrier U=1.5 eVthickness of the barrier d = 0.70 nmbias voltage V

Parameters roughly correspond to junctions composed of Fe electrodes and AlO_x barriers.



Calculations related to semiconductor junctions were also performed. Fermi energy $E_f = 0.1eV$; spin splitting of the electron band $2\Delta = 0.12eV$ height of the barrier U=0.1 eV; thickness of the barrier d = 2.0nm

In general the results show similar behavior.

Angular dependence of current induced spin torque



The dependence can be well described by a function $\sin \theta$. Confirmed by other theoretical approaches and experimental data It is a characteristic feature of MTJ. In metallic junctions due to spin accumulation a more complex dependence can be found.

$$T_{\parallel}^{r} = \left[(I_{P\uparrow} - I_{P\downarrow}) + (I_{AP\uparrow} - I_{AP\downarrow}) \right]_{\frac{\hbar}{4e}} \sin \theta$$

Theodonis et al..

$$\frac{dT_{\parallel}}{dV} = \left[(G_{\uparrow\uparrow} - G_{\downarrow\downarrow}) + (G_{\uparrow\downarrow} - G_{\downarrow\uparrow}) \right] \frac{\hbar}{4e} \sin\theta$$

Slonczewski

The torque magnitude increases with splitting parameter increasing

 $G_{\sigma\sigma'}$ the conductance related to electrons with spin σ in the left electrode and σ' in the right electrode

Angular dependence

Results of ab inito calculations (Heiliger, Stiles)

Torkance acting on the free layer is shown for different thicknesses of the Fe free layer

(fixed layer - 20ML, barrier - 6ML)

The torque is approximately confined to the interface.

The in-plane torkance is much larger than the out-of plane torkance

For ideal junctions both components are important, but the normal component is reduced due to thickness fluctuations



Angular dependence of spin torque

The torque magnitude depends on the junction parameters: splitting parameter, the height and the width of the barrier. When a barrier is low and wide the torque can change sign for low values of splitting parameter



The inversion of the torque sign results from the sign inversion of effective polarization

$$P_{i} = \frac{k_{i\uparrow} - k_{i\downarrow}}{k_{i\uparrow} + k_{i\downarrow}} \frac{\kappa^{2} - k_{i\uparrow}k_{i\downarrow}}{\kappa^{2} + k_{i\uparrow}k_{i\downarrow}}$$

Slonczewski $T^{r} / I \propto P$

$$k_{i\sigma}^2 = \frac{2mE_{i\sigma}}{\hbar^2}$$

$$\kappa^2 = \frac{2mU}{\hbar^2}$$

Bias dependence of in-plane torque





Bias behaviour depends on splitting parameter. In general, the in-plane component is asymmetric with respect to bias reversal. The torque acting on the collector (positive bias) is larger than acting on the source (negative bias). The asymmetry is more pronounced in systems with a strong splitting The normalized in-plane torque for positive bias achieves a plateau. The normalized torque for negative bias decreases and changes sign

 $T_{\parallel}^r / J \propto P_l$

Slonczewski

when the effective barrier height becomes low the polarization decreases and can be negative

Bias dependence of normal torque

 $\mathcal{G} = \pi/2$



The out-of plane component in symmetric junctions is symmetric and reveals a parabolic dependence on bias

 $T_{\perp}(V) = T_{\perp}^{0} + T_{\perp}^{1}V^{2}$

It is different form zero for unbiased systems and corresponds to the interlayer exchange coupling

Non-symmetrical junctions. Bias dependence



The in-plane torque acting on the right electrode essentially depends on the polarization of the left electrode, but it practically does not change with polarization of the right electrode what is well consistent with Slonczewski's predictions $T_{\parallel}^r/J \propto P_l$

The splitting of electron bands in both electrodes has similar influence on the normal torque. Due to different splitting of electron bands in both electrodes the normal torque is not symmetric with respect to bias reversal

MTJ with Semiconductor Electrodes



Fermi energy $E_f = 0.1 eV$; spin splitting of the electron band $2\Delta = 0.12 eV$ height of the barrier U=0.1 eV; thickness of the barrier d = 2.0 nm

Bias dependence (other theoretical approaches)



Bias dependence of in-plane torque

The interplay between spin currents in P and AP configurations determines bias dependence of the in-plane torque

Theodonis et al.

$$T_{\parallel}^{r} = \left[(I_{P\uparrow} - I_{P\downarrow}) + (I_{AP\uparrow} - I_{AP\downarrow}) \right] \frac{\hbar}{4e} \sin \theta$$

z component of spin current in P configuration (θ =0) depends in a linear way on bias voltage

z component of spin current in AP configuration ($\theta=\pi$) depends in a parabolic way on bias

Bias dependence Ab initio calculations for Fe/MgO/Fe

For samples with typical interfacial roughness the in-plane torque varies in a linear way with bias and normal torque shows a parabolic dependence



Bias dependence of torque components for an average free layer thickness of 19 monolayers

Heiliger, Stiles PRL 2008

Bias dependence. Comparison with experiment

Experiment – Kubota et al. (2008) CoFeB/MgO/CoFeB, spin torque diode effect

Theory – Theodonis et al. (2006)



The experimental curves are not reproduced for the same parameter set



The in-plane component is found to be asymmetric with respect to the bias reversal, for one bias polarization it can be negative. The out-of plane torque shows a parabolic dependence.

Bias dependence. Comparison with experiment Experiment – Kubota et al. (2008) CoFeB/MgO/CoFeB Theory – Xiao, Bauer, Brataas (2008) scattering matrix approach



The experimental curves are reproduced for the same parameter set in-plane torkance $A(dN_{\parallel}/dV)/\sin\theta$ (left) and out-of-plane torkance $A(dN'_{\perp}/dV)/\sin\theta$ (right). The following parameters are used in all fittings: $E_{\rm F} = 4.5 \text{ eV}, \Delta = 0.87E_{\rm F} \approx 3.9$ eV, $U_{\rm b} = 0.23E_{\rm F} \approx 1.0 \text{ eV}, \beta = 0.36; d = 1 \text{ nm}, \text{ cross section}$ area $A = 70 \text{ nm} \times 250 \text{ nm}, \text{ and } \theta = 137^{\circ}$ from Ref. 22.

Bias dependence. Experimental results



Bias dependence of spin transfer torkance

Voltage (mV)

2.0 kOe, 52*

400

200



-200

-400

-0.1

FIG. 8: (Color online) Fitting (curves) of the experimental data (dots) from Ref. 21. The following parameters are used in both fittings: $E_{\rm F} = 4.5 \text{ eV}, \Delta = 0.85 E_{\rm F} \approx 3.8 \text{ eV}, U_{\rm b} = 0.25 E_{\rm F} \approx 1.1 \text{ eV}, \beta = 0.43; d = 1.25 \text{ nm}, \text{ cross section area} A = 50 \text{ nm} \times 100 \text{ nm}, \text{ and } \theta = 71^{\circ} \text{ from Ref. 21}$ (Notice the sign convention in Ref. 21 is opposite to that in Ref. 22).

Bias dependence. Experimental results

The in-plane torkance shows a weak bias dependence and decreases when voltage is changed from V=-0.3V to 0.3 V. However, the strong asymmetry on the bias revealed by Kubota has not been found

Bias and Angular Dependence of Spin-Transfer Torque Tunnel Junctions

C. Wang¹, Y.-T. Cui¹, J. Z. Sun², R. A. Burhman¹, and D. C. Ralph¹ ¹Cornell University, Ithaca, New York 14853, USA ²IBM T. J. Watson Research Center, Yorktown Heights, New York 10598, USA



Bias dependence. Experimental data





Deac et al. (2008)

Normal torque shows a parabolic dependence In-plane torque shows a linear dependence but the slope changes at higher voltages and different constants are found for positive and negative bias

There is a good consistency of experimental and theoretical results for normal torque, but not for in-plane torque

Experimental results

Sankey et al. 2008 CoFeB/MgO/CoFeB

Spin-transfer-driven ferromagnetic resonance



Ratio of the normal torkance to in-plane torkance

Junction with a ferromagnetic layer of finite thickness



Junction with a ferromagnetic layer of finite thickness. Current induced torque



Quantum interference leads to oscillations of charge current in each spin channel

Torque components oscillate with thickness of ferromagnetic layer d_f

The average value of the current induced torque is roughly the same as in junction with two semi-infinite electrodes

Bias dependence



Bias dependence of in-plane torque is non-monotonic

The form of the behaviour depends on the film thickness.

Positions of resonance states in the well change with bias voltage and they strongly influence the charge and spin currents.

Oscillations can be seen when the layer is thin

Double Tunnel Junction

The enhancement of the tunneling current as well as of the spin torque can be obtained in the double junction due to formation of resonant states in the central layer

Parameters of the junction Fermi energy $E_f = 0.1eV$ spin splitting of the electron band $2\Delta = 0.12eV$ height of the barriers U=0.1 eVthickness of the barriers $d_b = 2.0nm$ thickness of the central layer d_c bias voltage V

The torque is equal to the spin current absorbed by the central layer

$$T_{\parallel} = \frac{\hbar}{2} \left(J_x^{sl} - J_x^{sr} \right) \qquad T_{\perp} = \frac{\hbar}{2} \left(J_y^{sl} - J_y^{sr} \right)$$



direction of tunnelling for the bias voltage V>0

Coherent tunnelling is assumed

Spin current densities are calculated in the central layer at left and right interfaces

Thickness dependence of the spin torque

The torque oscillates with the thickness of the central layer d_c . The resonant enhancement can be obtained for small d_c due the presence of resonant states formed in the central layer. Oscillations are damped for large d_c .

The in-plane torque in junctions with a thick central layer for a small bias voltage is larger in AP configuration, whereas the normal torque is larger in P configuration

 $T_{\parallel}^{cAP} > T_{\parallel}^{cP} - T_{\perp}^{cP} > T_{\perp}^{cAP}$

The torque acting on the central layer can be considered as a sum of two torques exerted by electrons tunnelling through the left and right barriers (in junctions with thick central layer).

 $T^c = T^l + T^r$

$$T_{\perp}^{lP} \approx T_{\perp}^{rP} \qquad T_{\perp}^{lAP} \approx -T_{\perp}^{rAP} \Longrightarrow T_{\perp}^{cAP} \approx 0$$



Angular dependence

The angular-dependence can be more complex than in a single junction. In general, $\sin\theta$ dependence is obtained for in-plane torque, but a maximum can be shifted for a certain thickness of central layer. The normal torque can vanish for noncollinear configuration. The sign of the normal torque depends on the central layer thickness





Angular dependence (Theodonis et al.)



The angular dependence of the normal torque varies with a thickness of central layer. For a certain thickness the normal torque vanishes in noncollinear configuration



The angular dependence of the torque varies with the bias voltage



Angular dependence (Theodonis et al.)

$$T_{\perp} = -\frac{\partial E_{ex}(\theta)}{\partial \theta}$$

Effective exchange coupling

$$E_{ex}(\theta) = -J_1 \cos \theta - J_2 \cos^2 \theta + \dots$$

 J_1 non equilibrium bilinear effective exchange coupling J_2 non equilibrium biquadratic effective exchange coupling

Interplay between bilinear and biquadratic terms determines the angular dependence of normal torque



Conclusions

Normal component of STT different from zero in MTJ Single Tunnel Junction

- Angular dependence of STT well described by a sine function
- Free-electron-like model leads to results consistent with other theoretical approaches (tight binding, scattering matrix approaches)
- In-plane component can show different bias dependence for different parameters
- Out-of-plane component is symmetric and reveals a parabolic-like dependence
- The full consistency of theoretical and experimental results is not reached yet
- STT shows an oscillatory dependence on the thickness of ferromagnetic layer which is a result of the quantum size effect

Double Tunnel Junction

- Torque components strongly depend on the thickness of the central layer
- The in-plane torque is smaller in P configuration than in AP one, whereas the opposite relation is observed for the normal component (for small voltage)

Thank you for attention

Wave function of tunnelling electron

the spinor in the *i*-th electrode (i=l,r) in the local reference frame takes the form for incident electron of spin σ

$$\Psi_{i\sigma}(y) = \begin{bmatrix} \psi_{i\sigma\uparrow} \\ \psi_{i\sigma\downarrow} \end{bmatrix} = \begin{bmatrix} A_{i\sigma\uparrow} \exp(ik_{i\uparrow}y) + B_{i\sigma\uparrow} \exp(-ik_{i\uparrow}y) \\ A_{i\sigma\downarrow} \exp(ik_{i\downarrow}y) + B_{i\sigma\downarrow} \exp(-ik_{i\downarrow}y) \end{bmatrix}$$

in the barrier

$$\Psi_{B\sigma}(y) = \begin{bmatrix} \Psi_{B\sigma\uparrow} \\ \Psi_{B\sigma\downarrow} \end{bmatrix} = \begin{bmatrix} C_{B\sigma\uparrow} Ai(Z) + D_{B\sigma\uparrow} Bi(Z) \\ C_{B\sigma\downarrow} Ai(Z) + D_{B\sigma\downarrow} Bi(Z) \end{bmatrix}$$
Ai, Bi -Airy functions
$$Z = f(y)$$

for electron of spin σ = \uparrow impinging from the left on the left barrier $\begin{array}{c} A_{l\uparrow\uparrow} = 1 \\ B_{r\uparrow\sigma} = 0 \end{array} \quad \begin{array}{c} A_{l\uparrow\downarrow} = 0 \\ B_{r\uparrow\sigma} = 0 \end{array}$

 $k_{i\sigma'}$ is the component of the wave vector normal to electrode/barrier interface,

$$\psi_{\sigma\downarrow B} = -\psi_{\sigma\uparrow r} \sin\left(\frac{\theta}{2}\right) + \psi_{\sigma\downarrow r} \cos\left(\frac{\theta}{2}\right) \qquad \psi_{\sigma\uparrow B} = \psi_{\sigma\uparrow r} \cos\left(\frac{\theta}{2}\right) + \psi_{\sigma\downarrow r} \sin\left(\frac{\theta}{2}\right)$$

The matching conditions at the barrier/right FM interface take the form $A_{i\sigma\uparrow}, B_{i\sigma\uparrow}, C_{Bj\sigma\uparrow}, D_{Bj\sigma\uparrow}, A_{i\sigma\downarrow}, B_{i\sigma\downarrow}, C_{Bj\sigma\downarrow}, D_{Bj\sigma\downarrow}$ From the matching conditions

$$\psi_{i\sigma} = A_{i\sigma} \exp(ik_{i\sigma}y) + B_{i\sigma} \exp(-ik_{i\sigma}y)$$

$$k_{i\sigma} = \sqrt{\frac{2m(E - E_{i\sigma}^{b})}{\hbar^{2}} - k_{\parallel}^{2}} = \frac{\sqrt{2m(\varepsilon_{\perp} - E_{i\sigma}^{b})}}{\hbar}$$

 $E_{i\sigma}^{b}$ denotes the electron band bottom for spin σ in the *i*-th layer, is the in-plane component of the wave vector, $\varepsilon_{\perp} = E - \hbar^{2} k_{\parallel}^{2} / 2m$ is the electron energy associated with its motion in the direction perpendicular to the layers, and *m* is the free electron mass.

Currents, torques, and polarization factors in magnetic tunnel junctions

 $TMR = (R_{AP} - R_P)/R_P = 2\iota/(1 - \iota) \qquad \iota = P_L P_R$ $T_L = -(\hbar \tau_L J_0/2e) \sin \theta, \quad \tau_L = P_R$



FIG. 3. Schematic effect of finite voltage on TMR, polarization, and torque coefficients illustrated by the toy free-electron model of a physically symmetric magnetic tunnel junction. Note that TMR is symmetric, but the other coefficients are not. The parameters are $\kappa_0 = 6.4k_-$, $k_+ = 10k_-$.

J. C. Slonczewski*

$$P_{i} = \frac{k_{i,+} - k_{i,-}}{k_{i,+} + k_{i,-}} \cdot \frac{\kappa_{0}^{2} - k_{i,+} k_{i,-}}{\kappa_{0}^{2} + k_{i,+} k_{i,-}}$$

$$k_{i,\sigma}^2 = 2mE_{i,\sigma}/\hbar^2$$
 and $\kappa_0^2 = 2mB/\hbar^2$



FIG. 2. Schematic junction potential for finite V. The shaded bar indicates the energy range of most of the tunneling electrons.

We simplify this model one step further and neglect the width of the shaded current band in Fig. 2. It is then clear

$$\begin{split} k_{L\sigma}^2 &= 2mE_{L\sigma} / \hbar^2 \quad \kappa_0^2 = 2mB / \hbar^2 \\ k_{R\sigma}^2 &= 2m(E_{R\sigma} + eV) / \hbar^2 \text{ and } \kappa_0^2 = 2m(B - eV) / \hbar^2, \end{split}$$

The relation $\tau_L = P_R$ can be obtained within the Bardeen transfer matrix formalism (with the additional assumption that polarization factor can be represented by a product of interfacial densities of states which can be justified for alumina barriers).

Total spin current density The total spin current density is calculated by taking the sum over all occupied states

$$J_{\mu}^{s}(y) = J_{\mu}^{sl \to r}(y) + J_{\mu}^{sr \to l}(y)$$

$$J_{\mu}^{sl \to r}(y) = \frac{4\pi^{2}m^{2}}{h^{4}} \sum_{\sigma'} \int_{E_{l\sigma'}}^{E_{r}} d\varepsilon_{\perp} \frac{(E_{r} - \varepsilon_{\perp})}{k_{l\sigma'}(\varepsilon_{\perp})} j_{\mu}^{s\sigma',l \to r}(y, \varepsilon_{\perp})$$

$$J_{\mu}^{sr \to l}(y) = \frac{4\pi^{2}m^{2}}{h^{4}} \sum_{\sigma'} \int_{E_{r\sigma'}}^{E_{r} - eV} d\varepsilon_{\perp} \frac{(E_{r} - eV - \varepsilon_{\perp})}{k_{r\sigma'}(\varepsilon_{\perp})} j_{\mu}^{s\sigma',r \to l}(y, \varepsilon_{\perp})$$

 $E_{\rm F}$ Fermi energy in the left electrode, $E_{l(r)\sigma}^{b}$ - position of the band bottom for the electrons of spin σ' in the left (right) electrode.

When the magnetization direction is non-uniform the spin currents carried by electrons moving from the left to the right and from the right to the left do not cancel and the net spin current appears with no bias applied. Normal spin currents flowing in opposite directions have the same values and signs for all energies in the spin split band $j_{\perp}^{sl \to r}(\varepsilon_{\perp})_{V=0} = j_{\perp}^{sr \to l}(\varepsilon_{\perp})_{V=0}$

Contributions to the in-plane spin currents flowing in opposite directions have the same values but they differ in signs, so the net current vanishes.

When the voltage is applied electrons with energy from the tunnel window contribute to the charge current and the spin current





Electrons which tunnel from the right electrode to the left with normal energy lying below the band bottom in the left electrode practically do not contribute to the normal torque in MTJ with metallic electrodes. Small contribution can be found for MTJ with semiconductor electrodes Chshiev (2008)



$$Q_{\perp}^{L(R)}(E)\Big|_{V>0} = Q_{\perp}^{R(L)}(E - eV)\Big|_{V<0}$$
$$Q_{\perp}(E)\Big|_{V>0} = Q_{\perp}(E - eV)\Big|_{V<0}.$$
$$Q_{\parallel}^{L}(E) = -Q_{\parallel}^{R}(E) \text{ for } E < \min\{f_{L}, f_{R}\},$$
$$f_{L}(E_{F}) = f_{R}(E_{F} \pm eV)$$

Fermi-Dirac distribution functions



The interplay between spin currents in P and AP configurations determines bias dependence of the torque

$$T_{\parallel}^{r} = \left[(I_{P\uparrow} - I_{P\downarrow}) + (I_{AP\uparrow} - I_{AP\downarrow}) \right]_{\frac{\hbar}{4e}} \sin \theta$$

z component of spin current in P configuration (θ =0) depends in a linear way on bias voltage

z component of spin current in AP configuration ($\theta=\pi$) depends in a quadratic way on bias Theodonis

In P configuration electrons with both spins tunnel through symmetric barriers, but heights of barriers are different. Currents in both spin channels vary linearly with bias voltage In AP configuration electrons with both spins tunnel through asymmetric barriers with the same average barrier height. Due to asymmetry of the barriers the linear terms cancel for both spin channels





The normalized torque components decrease with increasing barrier width, A different behavior is found for the dependence on the barrier height U. Now, the magnitude of both normalized torque components increases with increasing U which is a consequence of the fast decay of charge current

 $\theta = \pi / 2$

The normalized out-of-plane and in-plane components of the spin torque exerted on the right ferromagnetic electrode, calculated as a function of the barrier thickness (a) and barrier height (b).

Junction with a ferromagnetic layer of finite thickness. Current induced torque



Quantum interference leads to oscillations of the charge current in each spin channel. The periods can be expressed by half of the wavelength in the magnetic layer of electrons which tunnel perpendicularly to the interface.

$$d_{\sigma} = \lambda_{\sigma} / 2 = \pi / k_{\sigma} = \pi \hbar / \sqrt{2m(E_f + \hat{\sigma}\Delta + eV)}$$

Superposition of the oscillations in two spin channels leads to a complex dependence of the charge and spin currents and also of the torque on the layer thickness

Junction with a ferromagnetic layer of finite thickness



The dynamics of the free layer's magnetization under the influence of a spinpolarized current can be described by a modified Landau-Lifshitz-Gilbert equation

$$\begin{aligned} \frac{\partial \vec{m}}{\partial t} &= -\gamma \vec{m} \times \vec{H}_{eff} + \alpha \vec{m} \times \frac{\partial \vec{m}}{\partial t} + \frac{\gamma}{M_s Vol} T_{//} \vec{m} \times (\vec{m} \times \vec{m}_r) + \frac{\gamma}{M_s Vol} T_{\perp} (\vec{m} \times \vec{m}_r) \\ \vec{H}_{eff} &= \vec{H}_{ext} + \vec{H}_{anis} + \vec{H}_{ex} + \vec{H}_{magnetostatic} \end{aligned}$$

the peak width

$$\Delta f = \frac{\gamma}{2\pi} \alpha (4\pi M_s + 2H_{eff}) + 2\frac{\gamma}{2\pi} \frac{T_{//}}{M_s Vol}$$

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the resonance frequency

$$f = \frac{\gamma}{2\pi} \sqrt{\left(H_{eff} + \frac{T_{\perp}}{M_{s}Vol}\right) \left(H_{eff} + \frac{T_{\perp}}{M_{s}Vol} + 4\pi M_{s}\right)}$$

In red: values obtained from the ends mode data. In black:

values deduced from the centre mode data. Full (empty) symbols mark data

points obtained from measurements close to the P (AP) state. Blue lines: best fit



Two signals corresponding to two different precessional modes are investigated. Data for negative bias are obtained from measurements close to P states (at H=-250 Oe), while for positive voltages from signals around AP configuration (at H=200Oe). Values obtained from two signals at positive voltages are in good agreement (for normal component), while for negative voltages some differences can be seen.

Local torque strongly oscillates in the central layer



FIG. 4: (Color online) Angular dependence of $T_{i,\perp}$ (black squares) and $T_{i,\parallel}$ (red circles) for $N_c = 4$ AS in panels (a) and (b), and for $N_c = 7$ AS in panels (c) and (d), respectively. The local spin torque is evaluated on the first site in the central FM region next to the left FM/I interface at T = 5K and V = 0.1V.



FIG. 8: (Color Online)(a) Angular dependence of the *net* (a) field-like torque and (b) spin-transfer torque, for $N_c = 4$ and various values of bias.

effective exchange coupling energy

$$E_{XC}(\theta) = -J_1 \cos(\theta) - J_2 \cos^2(\theta) + \dots,$$

 J_1 non equilibrium bilinear effective exchange coupling J_2 non equilibrium biquadratic effective exchange coupling

Total torque

$$T = \sum_{i} T_{i}$$

 $T_{\perp} = -\partial E_{XC}(\theta)/\partial\theta,$

Tunnel magnetoresistance

$$TMR = \frac{R_{AP} - R_P}{R_P}$$

TMR effect applied in:

Magnetic field sensors in the read heads of magnetic hard disks drives Non-volatile random access memory

Magnetic switching driven by the spin transfer torque is much more efficient that switching driven by current induced magnetic field. This may enable the production of magnetic memory devices with much lower switching current and greater energy efficiency and greater device density than field switched devices



dc voltage is produced across the junction in a presence of small high frequency current. Voltage as a function of frequency shows a resonance which shape depends on the spin torque

$$V_{diode}(\omega) = f(T_{\parallel}, T_{\perp}, \omega, \theta, H)$$



Spin-torque diode spectra measured under various d.c. bias voltages.